

Number of Solutions to a Quadratic Equation

There is one more useful thing that can be determined from just a piece of the quadratic formula. The part under the root symbol in the formula is called the *discriminant*. The value of the discriminant can tell you the number of roots the quadratic has.

The discriminant, D , of a quadratic in the standard form $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

- If the discriminant is positive, the quadratic has 2 real solutions.
- If the discriminant equals 0, the quadratic has 1 real solution.
- If the discriminant is negative, the quadratic has no real solutions.

You saw earlier in this chapter how a question might ask about the number of solutions to a system of linear equations. When you see a similar question about a single quadratic, see if you can graph it or use the discriminant.

Take a look at a typical question.



 Mark for Review

19. If the equation $12x = k - 4x^2$ has exactly one real solution, what is the value of the constant k ?



Watch Us Crack It

Check out the Video Walkthroughs in your online Student Tools to watch a Princeton Review teacher work through this question step-by-step.

Here's How to Crack It

The question asks for the value of a constant in a quadratic equation. When a quadratic has exactly one real solution, the discriminant equals 0. First, put the quadratic in standard form, which is $ax^2 + bx + c$. Add $4x^2$ to both sides of the equation to get $4x^2 + 12x = k$, and then subtract k from both sides of the equation to get $4x^2 + 12x - k = 0$. Now that the quadratic is in standard form, $a = 4$, $b = 12$, and $c = -k$. Plug the values for a , b , and c into the discriminant, $D = b^2 - 4ac$, and set it equal to 0 to get $12^2 - (4)(4)(-k) = 0$. Simplify the left side of the equation to get $144 + 16k = 0$. Subtract $16k$ from both sides of the equation to get $144 = -16k$. Divide both sides of the equation by -16 to get $-9 = k$. The correct answer is -9 .



GROWTH AND DECAY

There's one more equation with an exponent that's tested frequently on the Digital SAT: the **growth and decay** formula. Real-world examples include population growth, radioactive decay, and compound interest, to name a few. The growth or decay can be a percent or a multiple, which changes what's inside the parentheses.

When the growth or decay rate is a percent of the total population:

$$\text{final amount} = \text{original amount} (1 \pm \text{rate})^{\text{number of changes}}$$

When the growth or decay is a multiple of the total population:

$$\text{final amount} = \text{original amount} (\text{multiplier})^{\text{number of changes}}$$

Let's see how this formula can make quick work of an otherwise tedious question.

 Mark for Review

20. Becca deposits \$100 into a bank account that earns an annual interest rate of 4%. If she does not make any additional deposits and makes no withdrawals, how long will it take her, in years, to increase the value of her account by at least 60%?

☐ A 12

☐ B 15

☐ C 25

☐ D 30

Here's How to Crack It

The question asks for the number of years it will take for Becca's account to reach a certain value. You could add 4% to the account over and over again

until you get to the desired amount, but that would take a long time. Knowing the formula will make it a lot easier. First, set up the equation with the things you know. The original amount is 100, and the rate is 4%, or 0.04. The account is increasing, so you add the rate, and you can put in “years” for the number of changes. The formula becomes

$$\text{final amount} = 100(1 + 0.04)^{\text{years}}$$

Now you need to figure out what you want the final amount to be. Translate the English to math: the value of her account (100) will increase (+) by 60 percent (0.6) of the current value ($\times 100$). This becomes $100 + (0.6)(100) = 100 + 60 = 160$. Now the formula is

$$160 = 100(1.04)^{\text{years}}$$

The answer choices represent the number of years Becca keeps her money in the account. Now you are all set to easily plug in the answers. Start with (B), so $\text{years} = 15$. Is $100(1.04)^{15} = 160$? Use your calculator to check, making sure to follow PEMDAS rules and do the exponent before you multiply by 100. The result is \$180.09. That is a bit too much money, so the answer will likely be (A), but let’s just check it. $100(1.04)^{12} = \$160.10$, which is at least \$160. The correct answer is (A).



Here’s an example of growth with a multiplier instead of a rate.



 Mark for Review

21. An invasive species was discovered to have a population of 2,100 individuals after 10 years of uninhibited growth. The equation $P = G(3)^{\frac{t}{10}}$ gives the number of individuals in the population, where t is

the number of years after the uninhibited growth began, P is the current number of individuals in the population, and G is the number of individuals in the population when the uninhibited growth began. What is the value of G ?

☐ A 30

☐ B 700

☐ C 2,100

☐ D 6,300

Here's How to Crack It

The question asks for a value in an equation. The equation is in the form of the growth and decay formula, and recognizing that leads to the next step. Label each piece of the formula and fill in the number given for each piece. The question states that P is the current number of individuals in the population, and that the invasive species had a population of 2,100 individuals, so $P = 2,100$. The question also states that 2,100 was the final amount after 10 years of uninhibited growth, and that t is the number of years after the uninhibited growth began, so $t = 10$. Plug those values into the formula and solve for G : $2,100 = G(3)^{\frac{10}{10}}$. Simplify the right side of the equation to get $2,100 = G(3)^1$, and then $2,100 = 3G$. Divide both sides of the equation by 3 to get $700 = G$. The correct answer is (B).



WHEN VALUES ARE ABSOLUTE

Absolute value is a measure of the distance between a number and 0. Since distances are always positive, the absolute value of a number is also always positive. The absolute value of a number is written as $|x|$.

When solving for the value of a variable inside the absolute value bars, it is important to remember that the variable could be either positive or negative. For example, if $|x| = 2$, then $x = 2$ or $x = -2$, as both 2 and -2 are a distance of 2 from 0.

Here's an example.



 Mark for Review

22. Which of the following is the value of $|y + z|$ if y and z are the solutions to the equation $|-4x - 2| = 6$?

☐ A -3

☐ B -2

☐ C 1

☐ D 3

Digital SAT Smoke and Mirrors

When you're asked to solve an equation involving an absolute value, it is very likely that the correct answer will involve the negative result. Why? Because the test-writers know that you are less likely to think about the negative result! Another way to avoid mistakes is to do all the math inside the absolute value symbols first, and then make the result positive.

Here's How to Crack It

The question asks for the value of an expression with an absolute value given an equation with an absolute value. Start with the equation and find the two solutions. Remember that the expression inside the absolute value symbols could be positive or negative and will still yield a positive result. Set that expression, $-4x - 2$, equal to 6 and -6 , and solve for the two solutions.

$$\begin{aligned}-4x - 2 &= 6 \\ -4x &= 8 \\ x &= -2\end{aligned}$$

and

$$\begin{aligned}-4x - 2 &= -6 \\ -4x &= -4 \\ x &= 1\end{aligned}$$

Thus, the solutions y and z are -2 and 1 . There's no way to know which is y and which is z , but it doesn't matter. Replace the variables in the expression $|y + z|$ with those values in either order, and calculate the result.

$$|-2 + 1| = |-1| = 1$$

If you're curious, try the other order for y and z to see that, because of the absolute value, it still works.

$$|1 + (-2)| = |1 - 2| = |-1| = 1$$

In either case the result is 1. The correct answer is (C).



Algebra Drill

Work these questions without your calculator using the skills you've learned so far. Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. For the inequality $3 > 9x - 3$, which of the following is a possible value of x ?

☐ A 0

☐ B $\frac{2}{3}$

☐ C 3

☐ D 6

 Mark for Review

2. For the equation $\sqrt{mx - 5} = x + 3$, the value of m is -3 . Which of the following contains all possible solutions to the equation?

☐ A $-3, 3$

☐ B -2

☐ C $-2, -7$

☐ D $3, 6$

 Mark for Review

3. The equation $\frac{ac}{20} = \frac{3d}{z}$ expresses the relationship among a , c , d , and z .

Which of the following equations expresses z in terms of a , c and d ?

☐ A $z = \frac{60d}{ac}$

☐ B $z = \frac{ac}{60d}$

☐ C $z = \frac{3ac}{20}$

☐ D $z = \frac{20ac}{3d}$

 Mark for Review

4. A student spends $4\frac{1}{2}$ hours each day working on history and science assignments. It takes the student $\frac{1}{4}$ of an hour to complete a history assignment and $\frac{1}{2}$ of an hour to complete a science assignment. Which of the following equations represents the number of history assignments, h , and science assignments, s , the student can complete each day?

☐ A $\left(h + \frac{1}{2}\right)\left(s + \frac{1}{4}\right) = \frac{9}{2}$

☐ B $\left(\frac{1}{2} + \frac{1}{4}\right)(h + s) = \frac{9}{2}$

☐ C $\frac{1}{4}h + \frac{1}{2}s = \frac{9}{2}$

☐ $\frac{1}{2}h + \frac{1}{4}s = \frac{9}{2}$

 Mark for Review

5. If $x + 6 > 0$ and $1 - 2x > -1$, which of the following values of x is NOT a solution?

☐ A -6

☐ B -4

☐ C 0

☐ D $\frac{1}{2}$

 Mark for Review

6. If $\frac{2x}{x^2+1} = \frac{2}{x+2}$, what is the value of x ?

☐ A $-\frac{1}{4}$

☐ B $\frac{1}{2}$

☐ C 0

☐ D 2

 Mark for Review

7. If the product of x and y is 76, and x is twice the square of y , which of the following pairs of equations could be used to determine the values of x and y ?

☐ A $xy = 76$
 $x = 2y^2$

☐ B $xy = 76$
 $x = (2y)^2$

☐ C $x + y = 76$
 $x = 4y^2$

☐ D $xy = 76$
 $x = 2y$

 Mark for Review

8. A tadpole that has just hatched from an egg weighs t grams. The equation $y = t(3)^w$ displays the weight y , in grams, of the tadpole w weeks after it hatches. If the tadpole reaches a weight of 9.72 grams at 5 weeks after hatching, what is the value of t ?

 Mark for Review

9. How many solutions exist to the equation $|x| = |2x - 1|$?

☐ A 0

☐ B 1

☐ C 2

☐ D 3

 Mark for Review

10. The sum of three numbers, a , b , and c , is 400. One of the numbers, a , is 40 percent less than the sum of b and c . What is the value of $b + c$?

☐ A 40

☐ B 60

☐ C 150

☐ D 250

ALGEBRA DRILL ANSWERS AND EXPLANATIONS

1. A

The question asks for a value in an inequality. To begin to isolate x , add 3 to both sides of the equation to get $6 > 9x$. Divide both sides of the equation by 9 to get $\frac{6}{9} > x$. Because 9 is positive, it is not necessary to flip the inequality sign. Reduce the fraction to get $\frac{2}{3} > x$. Only (A) is less than $\frac{2}{3}$. The correct answer is (A).

2. B

The question asks for all possible solutions to an equation. Since the question gives the value of m , the first step is to plug that value into the original equation to get $\sqrt{-3x-5} = x+3$. Now square both sides of the equation to remove the square root: $(\sqrt{-3x-5})^2 = (x+3)^2$ or $-3x-5$

$= x^2 + 6x + 9$. Now combine like terms. If you combine the terms on the right side of the equation, you can avoid having a negative x^2 term. The equation becomes $0 = x^2 + 9x + 14$. Factor the quadratic to find the roots: $0 = (x + 2)(x + 7)$. The possible solutions to the quadratic are -2 and -7 . Don't forget to plug these numbers back into the original equation to check for extraneous solutions. Begin by checking $x = -2$. When you do this, you get $\sqrt{(-3)(-2)-5} = (-2) + 3$, or $\sqrt{6-5} = 1$, or $\sqrt{1} = 1$, which is true. Now, check $x = -7$. Set it up as $\sqrt{(-3)(-7)-5} = (-7) + 3$, and start simplifying to get $\sqrt{21-5} = -4 = -4$. You can technically stop simplifying here, as there is a negative number on the right-hand side of the equals sign. Remember, when taking a square root with a radical provided, it will yield the positive root only. So -7 cannot be part of the solution set. Be very careful of (C), which is a trap answer. The correct answer is (B).

3. **A**

The question asks for an equation in terms of a specific variable. To begin to isolate z , cross-multiply to get $(20)(3d) = (ac)(z)$, which becomes $60d = (ac)(z)$. Divide both sides of the equation by ac to get $\frac{60d}{ac} = z$. The correct answer is (A).

4. **C**

The question asks for an equation that represents a specific situation. Translate the information in bite-sized pieces and eliminate after each piece. All of the answer choices equal $\frac{9}{2}$, which represents the $4\frac{1}{2}$ hours each day the student spends on both types of assignment. The left side of the equation must add up to that total time. One piece of information says that it *takes the student $\frac{1}{4}$ of an hour to complete a history assignment*, and another piece says that h represents the number of history assignments. The total time spent on history assignments can be represented by $\frac{1}{4}h$. Eliminate (D) because it multiplies h by $\frac{1}{2}$ instead of by $\frac{1}{4}$. Choice (A) includes the term $\frac{1}{4}h$ after FOILing, but it also includes hs . There is no reason to multiply the number of history assignments by the number of science assignments, so eliminate (A). Choice (B) includes the term $\frac{1}{2}h$ after FOILing, but h should only be multiplied by $\frac{1}{4}$. Eliminate (B). Choice (C) also correctly multiplies the time per science assignment by the number of science assignments to get $\frac{1}{2}s$. The correct answer is (C).

5. A

The question asks for the value of x that is not a solution. Solve the first inequality by subtracting 6 from each side so that $x > -6$. You are looking for values that won't work for x , and x cannot equal -6 . Therefore, the answer must be (A). Just to be sure, solve the next inequality by subtracting 1 from each side to get $-2x > -2$. Divide by

-2, remembering to switch the sign because you are dividing by a negative number to get $x < 1$. The values in (B), (C), and (D) fit this requirement as well, so they are values for x and not the correct answer. The correct answer is (A).

6. **B**

The question asks for the value of x . To solve this equation, use cross-multiplication to get $(2x)(x + 2) = (x^2 + 1)(2)$. Expand the equation to get $2x^2 + 4x = 2x^2 + 2$. Once you combine like terms, the result is $2x^2 - 2x^2 + 4x = 2$ or $4x = 2$. Solve for x by dividing both sides by 4 to get $x = \frac{1}{2}$. The correct answer is (B).

7. **A**

The question asks for a pair of equations to represent the situation. Translate each statement, piece by piece. The first part tells you that “the product of x and y is 76.” Since *product* means multiplication, the first equation must be $xy = 76$, so you can eliminate (C). The second part says that “ x is twice the square of y ,” which translates to $x = 2y^2$, so eliminate (B) and (D), and (A) is the only choice left. Notice that only the y needs to be squared, which is why (B) is wrong. The second equation for (B) would be written as “the square of twice y ,” which is not what the question states. The correct answer is (A).

8. **0.04**

The question asks for a value given a specific situation. The weight of the tadpole is increasing by a multiple over time, so write down the growth and decay formula. That formula is *final amount* = (*original amount*)(*multiplier*)^{*number of changes*}. The equation in the question

already shows that 3 is the *multiplier*. The question states that *the tadpole reaches a weight of 9.72 grams at 5 weeks after hatching*, so 9.72 is the *final amount* and 5 is the *number of changes*. Plug in these values and solve for t . The equation becomes $9.72 = t(3)^5$. Start with the exponent on the right side of the equation to get $9.72 = t(243)$. Divide both sides of the equation by 243 to get $0.04 = t$. The correct answer is 0.04, which can also be entered in the fill-in box as .04 or a fractional equivalent.

9. C

The question asks for the number of solutions to an equation. If $|x| = |2x - 1|$, either $x = 2x - 1$ or $-x = 2x - 1$. The solutions to these equations are 1 and $\frac{1}{3}$, respectively. However, the only thing you need to recognize is that the equation has two different solutions. Another option is making a quick graph on a calculator, which will show you that there are two solutions and what those solutions are. Either way, the correct answer is (C).

10. D

The question asks for the value of $b + c$. This is a system of equations question in disguise. First, locate a piece of information in this question that you can work with. “The sum of three numbers, a , b , and c , is 400,” seems very straightforward. Write the equation $a + b + c = 400$. Now the question tells you that “one of the numbers, a , is 40 percent less than the sum of b and c .” Translate this piece by piece to get $a = (1 - 0.4)(b + c)$, or $a = 0.6(b + c)$. Distribute the 0.6 to get $a = 0.6b + 0.6c$. Arrange these variables so they line up with those in the

first equation as $a - 0.6b - 0.6c = 0$. To solve for $b + c$, stack the equations and multiply the second equation by -1 :

$$\begin{array}{r} a + b + c = 400 \\ -1(a - 0.6b - 0.6c) = 0(-1) \end{array}$$

Now solve:

$$\begin{array}{r} a + \quad b + \quad c = 400 \\ -a + 0.6b + 0.6c = 0 \\ \hline 1.6b + 1.6c = 400 \end{array}$$

Simplify by dividing both sides by 1.6 to get $b + c = 250$. The correct answer is (D).

Summary

- Don't "solve for x " or "solve for y " unless you absolutely have to. (Don't worry; your math teacher won't find out.) Instead, look for direct solutions to Digital SAT questions. Math section questions rarely require time-consuming computations or endless fiddling with big numbers. There's almost always a trick—if you can spot it.
- If a question contains an expression that can be factored, factor it. If it contains an expression that already has been factored, multiply it out.
- To solve systems of equations, simply add or subtract the equations. If you don't have the answer, look for multiples of your solutions. When the question asks for a single variable and addition and subtraction don't work, try to make something disappear. Multiply the equations to make the coefficient(s) of the variable(s) you don't want go to zero when the equations are added or subtracted.
- If a question asks for the number of solutions, the graphing calculator can help, but it's also useful to know the rules.
 - Two linear equations have *no solution* when they have the same coefficient on the variable but different constants. This makes the lines parallel.
 - Two linear equations have *exactly one solution* when a single (x, y) point is a solution to both equations. The two lines intersect once.
 - Two linear equations have *infinitely many solutions* when they have the same coefficient on the variable and the same constants. This makes them the same line.
- Some Digital SAT questions require algebraic manipulation. Use tricks when you can, but if you have to manipulate the equation, take your time and work carefully to avoid unnecessary mistakes. You don't get partial credit for getting the question mostly correct.

- When working with inequalities, don't forget to flip the sign when you multiply and divide by negative numbers.
- When asked to create an equation, start with the most straightforward piece of information. You can also use the equations in the answer choices to help you narrow down the possibilities for your equation. Eliminate any answers in which an equation doesn't match your equation.
- When a question asks for an extraneous solution, first solve the equation, and then plug the answers back into the equation. If the equation is not true when solved with the solution, then that solution is extraneous.
- When solving quadratic equations, you may need to use FOIL or factor to get the equation into the easiest form for the question task. Don't forget about the common equations that the test-writers use when writing questions about quadratics.
- To solve for the roots of a quadratic equation, set it equal to zero by moving all the terms to the left side of the equation, then factor and solve, or use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When solving for the sum or product of the roots, you can also use these formulas:

- sum of the roots: $-\frac{b}{a}$
- product of the roots: $\frac{c}{a}$

- The discriminant of a quadratic in the form $ax^2 + bx + c = 0$ is the value of $b^2 - 4ac$. If this value is positive, there are 2 real roots; if it is 0, there is 1 real root; if it is negative, there are no real roots. A graphing calculator can also help with questions about the number of solutions to a quadratic equation.
- When a question is about exponential growth or decay, use the following formula:

$$\text{final amount} = (\text{original amount})(1 \pm \text{rate})^{\text{number of changes}}$$

If the change is by a multiple instead of a rate, use the same formula with *multiplier* inside the parentheses.

- The absolute value of a number is its distance from zero; distances are always positive. When working inside the $| |$, remember to consider both the positive and the negative values of the expression. Also remember that $| |$ work like $()$; you need to complete all the operations inside the $| |$ before you can make the value positive.

Chapter 22

Other Digital SAT Algebra Strategies

Now that you're familiar with the basics of algebra, it's time to learn how to avoid using algebra on the Digital SAT. Yes, you read that correctly. Algebra questions on the Digital SAT are filled with traps carefully laid by the test-writers, so you need to know how to work around them. This chapter gives you the strategies you need in order to turn tricky algebra questions into simple arithmetic.

PRINCETON REVIEW ALGEBRA—AKA HOW TO AVOID ALGEBRA ON THE DIGITAL SAT

Now that you've reviewed some basic algebra, it's time to find out when and how NOT to use it on the Digital SAT. The Digital SAT is not your math class at school, and all that matters is the correct answer. So we're going to show you how to avoid doing algebra on the Digital SAT whenever possible. Even if you love algebra, using it all the time on the Digital SAT can actually hurt your score, and we don't want that.

We know it's difficult to come to terms with this. But if you use only algebra on the Digital SAT, you're doing exactly what the test-writers want you to do. You see, when the test-writers design the questions on the Digital SAT, they expect the students to use algebra to solve them. Many Digital SAT problems have built-in traps meant to take advantage of common mistakes that students make when using algebra. But if you don't use algebra, there's no way you can fall into those traps.

Plus, when you avoid algebra, you add one other powerful tool to your tool belt: you can use the built-in calculator or your own! Calculators can help with algebra, but they're designed first and foremost for arithmetic. Our goal, then, is to turn algebra on the Digital SAT into arithmetic. We do that using techniques we call Plugging In and Plugging In the Answers (PITA).

Your Best Friend

Plugging In allows you to use your calculator on many of the algebra questions that show up on the Digital SAT.

PLUGGING IN THE ANSWERS (PITA)

Algebra uses letters to stand for numbers. You don't go to the grocery store to buy x eggs or y gallons of milk. Most people



think about math in terms of numbers, not letters that stand for numbers.

You should think in terms of numbers on the Digital SAT as much as possible. On many Digital SAT algebra questions, even very difficult ones, you will be able to find the correct answer without using any algebra at all. You will do this by working backward from the answer choices instead of trying to solve the problem using your standard math-class methods.

Plugging In the Answers is a useful technique for solving word problems in which the answer choices are all numbers. Using this powerful technique can solve many algebra problems on the Digital SAT simply and quickly.

In algebra class at school, you solve word problems by using equations. Then, you check your solution by plugging in your answer to see if it works. Why not skip the equations entirely by simply checking the four possible solutions on the multiple-choice questions? One of these is the correct answer. You don't have to do any algebra, you will seldom have to try more than two choices, and you will never have to try all four. Use PITA for questions that ask for a specific amount.

Here's an example of using PITA instead of writing equations.

 Mark for Review

1. On a certain assignment, student X takes 20 seconds per grammar question and 35 seconds per punctuation question. On the same assignment, student Y takes 30 seconds per grammar question and 55 seconds per punctuation question. It takes 310 seconds for student X to complete the assignment and 480 seconds for student Y to complete the assignment. If those are the only two types of questions on the assignment, how many punctuation questions are on the assignment?

A 5

B 6

C 7

D 8

Here's How to Crack It

The question asks for a value given a specific situation. Sure, you could make up some variables, translate words into equations, and use skills from the previous chapter to solve the system of equations, but that's what you would do in math class at school. On the Digital SAT, **there's a better way**. Notice that the question asks for a single, specific value: the number of punctuation questions on the assignment. Each answer choice also has a single, specific value. One of those values has to be the answer to the question, so start with the answers and plug them into the question.

Before you plug in an answer, make sure you know what the answers represent. The question asks for the number of punctuation questions, so rewrite the answers on your scratch paper and label them “# of punct Qs” or something similar.

Representation

Make sure you know what the numbers in the answer choices represent. Be sure to label them!

Now it's time to start working the steps of the problem. But first, notice that the answer choices are in numerically ascending order. The test-writers like to keep their questions organized, so they will always put the answers in order on this kind of question. You can use that to your advantage by starting with one of the middle answer choices. If it ends up being too big

or small, you should be able to eliminate more than one answer. If the first answer you plug in happens to work, you can just stop there. Try (B) first.

Look at (B) and ask yourself, “If the number of punctuation questions is 6, what’s the next thing I can figure out?” In this case, you could figure out how much time each student spends on punctuation questions. The question states that student X takes *35 seconds per punctuation question*, so student X will take $(6)(35) = 210$ seconds to finish the punctuation questions. It takes student X a total of 310 seconds to complete the assignment, so that leaves $310 - 210 = 100$ seconds for the grammar questions. The question also states that *student X takes 20 seconds per grammar question*, so there are $\frac{100}{20} = 5$ grammar questions.

Next, do the same thing for student Y, who takes 55 seconds per punctuation question. If there are 6 punctuation questions, that’s $(6)(55) = 330$ seconds. Student Y takes 480 seconds to complete the assignment, so that leaves $480 - 330 = 150$ seconds for the grammar questions. The question states that *student Y takes 30 seconds per grammar question*, so there are $\frac{150}{30} = 5$ grammar questions.

Take a look at the numbers on your scratch paper. Did both students complete 6 punctuation questions? Yes, because that’s the number you plugged in. Did both students complete 5 grammar questions? Yes, they did. Are you done? Yes, you are!

If you had started with a different number, somewhere along the way the math wouldn't have worked. You would end up with a fractional number of questions, or the two students wouldn't have the same number of each question type. When that happens, cross out the answer and try another one. In some cases, you'll be able to tell whether the number you tried was too big or too small, which helps you eliminate more answers and choose which one to try next. That's not necessary here because the first number you tried worked. The correct answer is (B).

Here are the steps for solving a problem using the PITA approach:

To solve a problem by plugging in the answers:



1. Rewrite the answer choices on your scratch paper and label them.
2. Starting with one of the middle answer choices, work the steps of the problem.
3. Look for something in the question that tells you what must happen for the answer to be correct.
4. When you find the correct answer, STOP.

You've seen how PITA gets you through a word problem quickly, efficiently, and accurately. But the technique also works on questions that aren't word problems but still ask for a specific value and have numbers in the answer choices. Here's an example.

$$\begin{aligned}2x + y &= 6 \\7x + 2y &= 27\end{aligned}$$

2. The system of equations above is satisfied by which of the following ordered pairs (x, y) ?

☐ A $(-5, 4)$

☐ B $(4, -2)$

☐ C $(5, 4)$

☐ D $(5, -4)$



Which Way?

Sometimes, it's hard to tell if you need a larger number or a smaller number if the first answer you tried didn't work. Don't fret. Just pick a direction and go! Spend your time trying answers rather than worrying about going in the wrong direction.

Here's How to Crack It

The question asks for the coordinates of the point that satisfies the system of equations. When you feel the urge to do a whole lot of algebra, it is a good time to check whether it would be possible to just plug in the answers instead. In this case, trying your answer choices will be not only effective but also incredibly fast.

Start by rewriting the answer choices on your scratch paper and labeling them “ (x, y) .” It doesn't seem like you will be able to tell whether to move up or down this time, as the ordered pairs don't really have an ascending or

descending order, but start in the middle anyway. Even if you end up trying three of the four, you will be saving time by plugging in the answers instead of solving.

Starting with (B) gives you 4 for x and -2 for y . Try that out in the first equation: $2(4) + (-2) = 6$. That matches the first equation, so this is a possibility. Try it out in the second equation: $7(4) + 2(-2) = 24$. That does not match the second equation, so you can eliminate (B).

Try out (C) next. If $x = 5$ and $y = 4$, then $2(5) + 4 = 14$, and you wanted it to be 6, so you can eliminate this answer choice as well.

Move on to (D). That would give you $2(5) + (-4) = 6$. So far so good! Try the second equation to see if this choice satisfies both: $7(5) + 2(-4) = 27$. This works, so the correct answer is (D).



You may recall that we covered questions like this in the last chapter. It is important to know how to solve these, in case a question like this comes up on a fill-in question. When you have answers available to you, though, don't be afraid to use them!

One last thing about PITA: here's how to determine whether you should use this approach to solve a problem.

Three ways to know that it's time for PITA:

1. There are numbers in the answer choices.
2. The question asks for a specific amount. Look for phrases like "the number of," "what was," or "how many."

3. You have the urge to write an algebraic equation to solve the problem.

Here's an example of using PITA instead of writing equations.

 Mark for Review

3. A bakery sold exactly 85% of the cupcakes it baked on Tuesday. Which of the following could be the total number of cupcakes baked on Tuesday?

☐ A 130

☐ B 140

☐ C 145

☐ D 150

Here's How to Crack It

The question asks for a possible total number of cupcakes. Is your first reaction that there isn't nearly enough information here to start on this question? That makes it a great opportunity to plug in the answers! Start with one of the middle answer choices and test it out. Sometimes, even if you can't see how a question works ahead of time, it starts to make a lot more sense once you plug real numbers into it.

Choice (C) is 145, but 145 what? Read the question very carefully. The question asks for the total number of cupcakes baked on Tuesday, so rewrite the answer choices and label them "Total."

Next, work your way through the problem. If 145 is the total number of cupcakes baked on Tuesday, the number the bakery sold on Tuesday is 85% of 145, or 123.25. Have you ever bought 0.25 cupcakes at a bakery? It would be really weird for a bakery to sell fractions of cupcakes, so this answer could not be the total number baked on Tuesday.

In this particular question, it is hard to tell whether you should try bigger or smaller numbers next, but you have learned two things from your first attempt: you can get rid of (C), and the correct answer will be the one that gives you a whole number of cupcakes. Instead of spending time trying to predict which direction to go for the answer, just get to work on the other answer choices.

Try (B) next. If the bakery baked 140 cupcakes on Tuesday, it sold 85% of 140, or 119. Is there anything wrong with selling 119 cupcakes? No! Since the bakery sold only whole cupcakes, the correct answer is (B).



Here's another example of how PITA turns algebra into arithmetic.



 Mark for Review

4. For what value of x is $|2x + 3| + 5 = 0$?

☐ A -4

☐ B 0

☐ C 4

☐ D There is no such value of x .

Here's How to Crack It

The question asks for the value of x . Although we covered it in the last chapter, solving algebraically on an absolute value question can be treacherous. There are so many ways to go wrong with those signs! Luckily, this absolute value question comes complete with answer choices, so we can simply plug in the answers to get a solution. As always, start by rewriting the answer choices on your scratch paper and labeling them. In this case, the label is simply “ x .”

Start with (C). When you put 4 in for x , you get $|2(4) + 3| + 5 = 0$, or $16 = 0$. This is clearly not true, so cross off (C) and move on to (B). If x is 0, then the original equation becomes $|2(0) + 3| + 5 = 0$ or $8 = 0$, so you can eliminate (B) as well. Next, try (A): $|2(-4) + 3| + 5 = 0$ could be rewritten as $|-8 + 3| + 5 = 0$, or $|-5| + 5 = 0$. As long as you remember that the absolute value of a number is always positive, it is clear that this gives you $5 + 5 = 0$. Since this is also clearly untrue, eliminate (A). Apparently, there is no such value of x ! The correct answer is (D).

SOLVING RATIONAL EQUATIONS

A rational equation is basically an equation in which one (or more) of the terms is a fractional one. Rational equations look scary, but there are very simple ways of solving them. All in all, the test-writers can't get too messy here, so they will keep the math nice and tidy. Try the following question.

 Mark for Review

$$x - 3 = \frac{5}{x - 3}$$

5. Which of the following is a possible value of $x - 3$ in the equation

☐ A $\sqrt{5}$

☐ B 5

☐ C $3 + \sqrt{5}$

☐ D 25



Watch Us Crack It

Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for a possible value of $(x - 3)$ in an equation with fractions and binomials. You could use your skills with quadratics from the last chapter, but it's much simpler to use PITA! After all, there are numbers in the answers and the question asks for a specific value, so it sure looks like a PITA question. This approach will also avoid the issue of extraneous solutions.

Follow the PITA approach: start by rewriting the answers on your scratch paper and labeling them as " $x - 3$ " and then start with one of the middle answer choices. The value in (C) looks complicated, so start with (B), 5. When $x - 3 = 5$, the equation becomes

$$5 = \frac{5}{5} \text{ or } 5 = 1$$

This is not true, so eliminate (B). Plugging in 25 would make the fraction too small, so ballpark out (D). Try the next simplest answer, (A). When $x - 3 = \sqrt{5}$, the equation becomes

$$\sqrt{5} = \frac{5}{\sqrt{5}}$$

Multiply both sides by $\sqrt{5}$ to get $5 = 5$, or use a calculator to find out that the two sides are equal. This makes the equation accurate, so $\sqrt{5}$ is a possible value of $x - 3$. The correct answer is (A).

PLUGGING IN YOUR OWN NUMBERS

Plugging In the Answers enables you to find the answer to questions whose answer choices are all numbers. What about questions that have answer choices containing variables? On these questions, you will usually be able to find the answer by plugging in your own numbers.

Plugging In is easy. It has three steps:



1. Pick numbers for the variables in the question.
2. Use your numbers to find an answer to the questions. Highlight your answer.
3. Plug your number(s) for the variable(s) into the answer choices and eliminate choices that don't equal the answer you found in Step 2.

The Basics of Plugging In Your Own Numbers

This sort of Plugging In is simple to understand. Here's an example.

 Mark for Review

6. Which of the following is equivalent to $\frac{3}{5} + \frac{y}{1-2y}$?

☐ A $\frac{3+y}{6-2y}$

☐ B $\frac{3-y}{5-10y}$

☐ C $\frac{2y}{5-10y}$

☐ D $\frac{3y}{5-10y}$

Here's How to Crack It

The question asks for an equivalent form of an expression. Rather than do complicated algebra, try plugging in. First, pick a number for y . Pick something easy to work with, like 2. On your scratch paper, write $y = 2$ so you won't forget. Then plug in 2 everywhere there's a y in the expression to get $\frac{3}{5} + \frac{2}{1-2(2)}$, which becomes $\frac{3}{5} + \frac{2}{1-4}$, then $\frac{3}{5} + \frac{2}{-3}$, and then $\frac{3}{5} - \frac{2}{3}$. Either find a common denominator and subtract the fractions by hand to get $\frac{9}{15} - \frac{10}{15} = -\frac{1}{15}$ or use a calculator to get the decimal equivalent of $-0.\overline{06}$. In either form, this is your target value. Write it on the scratch paper and circle it.

Next, plug $y = 2$ into the answer choices and eliminate ones that do not equal the target value. Rewrite the answers one at a time with 2 in place of y .

$$\text{A) } \frac{3+2}{6-2(2)} = \frac{5}{6-4} = \frac{5}{2}$$

Not the target; eliminate!

$$\text{B) } \frac{3-2}{5-10(2)} = \frac{1}{5-20} = \frac{1}{-15} = -\frac{1}{15}$$

Matches the target; keep but check the rest just in case.

$$\text{C) } \frac{2(2)}{5-10(2)} = \frac{4}{5-20} = \frac{4}{-15} = -\frac{4}{15}$$

Not the target; eliminate!

$$\text{D) } \frac{3(2)}{5-10(2)} = \frac{6}{5-20} = \frac{6}{-15} = -\frac{6}{15}$$

Not the target; eliminate!

Only (B) matches the target value, so it's the correct answer.



Here's another example.



 Mark for Review

7. During a special sale at a furniture store, Erica bought a floor lamp at a 10% discount. She paid a total of t dollars, which included the discounted price of the floor lamp and a 6% sales tax on the

discounted price. In terms of t , what was the original price of the floor lamp?

☐ A $\frac{t}{0.96}$

☐ B $(0.9)(1.06)t$

☐ C $\frac{t}{(0.9)(1.06)}$

☐ D $0.96t$

Get Real

Trying to imagine how numbers behave in the abstract is a waste of time. So, if the question says that Tina is x years old, why not plug in your own age? That's real enough. You don't have to change your name to Tina.

Here's How to Crack It

The question asks for an expression to represent the situation. This could be a pretty tricky algebra question, but if you read the question carefully and plug in easy numbers, it will be a breeze.

Start at the beginning. When Erica bought that floor lamp on sale, what did you really wish you knew? It would be very helpful to start this problem knowing the original price of the floor lamp. So, start plugging in there. Plug in a number that you know how to take a percentage of, like 100. Write down “original = 100” and move on to the next step of the problem. Erica got a 10% discount, so take 10% of the original price. That means she got a \$10 discount, and the discounted price of her floor lamp was \$90. Write that down and move on to the sales tax. If you read carefully, it is clear that the sales tax is 6% of the discounted price. So, you need to take 6% of the \$90 discounted price, or \$5.40. To get her total, add the \$5.40 of

tax to the \$90 for the discounted floor lamp, and you get \$95.40. This is where the careful reading comes in. The variable t in this question is supposed to be the total amount she paid, so make sure that you label this “ $t = \$95.40$.”

Next, read the last sentence of the question again to be sure you know which of the answers is your target answer. The question asks for the original price of the floor lamp, so circle the number you plugged in for the original price. Your target answer is 100.

On to the answer choices! When you put \$95.40 in for t in (A), you get 99.375. This is not your target answer, so you can eliminate (A). Choice (B) gives you 91.0116, so that will not work either. Plugging \$95.40 into (C) yields the target of 100, so hang on to it while you check (D) just in case. When you plug \$95.40 into (D), you get 91.584. Since that does not match your target, you can eliminate (D). The correct answer is (C).



Which Numbers?

Although you can plug in any number, you can make your life much easier by plugging in “good” numbers—numbers that are simple to work with or that make the problem easier to manipulate. Picking a small number, such as 2, will usually make finding the answer easier. If the question asks for a percentage, plug in 100. If the question has to do with minutes, try 30 or 120.

**Be Good**

“Good” numbers make a problem less confusing by simplifying the arithmetic. This is your chance to make the Digital SAT easier for you.

Except in special cases, you should avoid plugging in 0 and 1; these numbers have weird properties. Using them may allow you to eliminate only one or two choices at a time. You should also avoid plugging in any number that appears in the question or in any of the answer choices. Using those numbers could make more than one answer match your target. If more than one answer choice matches your target, plug in a new number and check those answer choices. You may have to plug in more than once to eliminate all three incorrect answers.

Many times you’ll find that there is an advantage to picking a particular number, even a very large one, because it makes solving the problem more straightforward.

Here’s an example.

 Mark for Review

8. If 60 equally priced downloads cost x dollars, then how much do 9 downloads cost?

$\frac{20}{3x}$

$$\boxed{\text{B}} \frac{20x}{3}$$

$$\boxed{\text{C}} 60x + 9$$

$$\boxed{\text{D}} \frac{3x}{20}$$

Here's How to Crack It

The question asks for the cost of 9 downloads. Since the question is asking you to arrive at a number (how much 9 downloads cost) in terms of variable x , try plugging in. Should you plug in 2 for x ? You could, but plugging in 120 would make the math easier. After all, if 60 downloads cost a total of \$120, then each download costs \$2. Write $x = 120$ on your scratch paper.

If each download costs \$2, then 9 downloads cost \$18. Write an 18 and circle it. You are looking for the answer choice that works out to 18 when you plug in \$120 for x . Try each choice:

$$\text{A) } \frac{20}{3(120)} \neq 18$$

$$\text{B) } \frac{20(120)}{3} \neq 18$$

$$\text{C) } 60(120) + 9 \neq 18$$

$$\text{D) } \frac{3(120)}{20} = 18$$

That last one matches the target answer, so the correct answer is (D).



Let's try another example.

 Mark for Review

9. A watch loses x minutes every y hours. At this rate, how many hours will the watch lose in one week?

☐ A $7xy$

☐ B $\frac{5y}{2x}$

☐ C $\frac{14y}{5x}$

☐ D $\frac{14x}{5y}$



Watch Us Crack It

Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for the number of hours lost in one week and gives information about minutes. This is an extremely difficult question for students who try to solve it using math-class algebra. You'll be able to find the answer easily, though, if you plug in carefully.

What numbers should you plug in? As always, you can plug in anything. However, if you think just a little bit before choosing the numbers, you can make the question easier to understand. There are three units of time—

minutes, hours, and weeks—and that’s a big part of the reason this question is hard to understand. If you choose units of time that are easy to think about, you’ll make the question easier to handle.

Start by choosing a value for x , which represents the number of minutes that the watch loses. You might be tempted to choose $x = 60$ and that would make the math pretty easy. However, it’s usually not a good idea to choose a conversion factor such as 60, the conversion factor between minutes and hours, for plugging in. When a question deals with time, 30 is usually a safer choice to avoid having multiple answers work the first time you plug in. So, write down $x = 30$.

Next, you need a number for y , which represents the number of hours. Again, you might be tempted to use $y = 24$, but that’s the conversion factor between hours and days. Therefore, $y = 12$ is a safer choice. Write down $y = 12$.

Now, it’s time to solve the problem to come up with a target. If the watch loses 30 minutes every 12 hours, then it loses 60 minutes every 24 hours. Put another way, the watch loses an hour each day. In one week, the watch will lose 7 hours. That’s your target, so be sure to circle it.

Now, you just need to check the answer choices to see which one gives you 7 when $x = 30$ and $y = 12$.

A) $7xy = 7(30)(12) = \text{Something too big!}$ Cross it off.

B) $\frac{5y}{2x} = \frac{5(12)}{2(30)} = \frac{60}{60} = 1$. Also wrong.

C) $\frac{14y}{5x} = \frac{14(12)}{5(30)} = \frac{168}{150} = \frac{28}{25}$. Cross it off.

D) $\frac{14x}{5y} = \frac{14(30)}{5(12)} = \frac{420}{60} = 7$. Choose it!

The correct answer is (D).

MEANING IN CONTEXT

Some questions, instead of asking you to come up with an equation, just want you to recognize what a part of the equation stands for. These questions will look like algebra, with variables and equations, but they're often more about reading carefully, working in bite-sized pieces, and using your pencil.

First things first, though, you want to think about your POOD. Do these questions make sense to you, or are they a little confusing? Do you usually get them right quickly, or do they take up too much time? You can always use the Mark for Review tool to flag a meaning in context question to come back to later after you get some easier, quicker questions right.

If the question does fit your POOD, read carefully, write down labels for the parts of the equation, and use POE to get rid of any answer choices that don't match your labels.

Here's an example.

 Mark for Review

10. There are m boxes of red marbles and n boxes of blue marbles in a crate, and the crate contains a total of 204 marbles. If this situation is represented by the equation $12m + 7n = 204$, what does the number 12 most likely represent?

☐ A The total number of red marbles

☐ B The total number of blue marbles

☐ C The number of red marbles in each box

☐ D The number of blue marbles in each box

Here's How to Crack It

The question asks for the meaning of a value in context. Start by reading the final question, which asks for the meaning of the number 12. Then label the parts of the equation with the information given. The question states that *there are m boxes of red marbles*, so label m as “# of boxes of red marbles.” Similarly, label n as “# of boxes of blue marbles.” The question also states that *the crate contains a total of 204 marbles*, so label 204 as “total # of marbles.” The equation is now labeled as follows:

$$12(\text{\# of boxes of red marbles}) + 7(\text{\# of boxes of blue marbles}) = (\text{total \# of marbles})$$

Next, use Process of Elimination to get rid of answer choices that are not consistent with the labels. Since 12 is multiplied by the number of boxes of red marbles, it has something to do with the red marbles. Eliminate (B) and (D) because they refer to blue marbles. Since 12 is multiplied by the number of boxes of red marbles, it must represent a value per box, not a total value. Keep (C) because it is consistent with this information, and eliminate (A) because it refers to a total number. The correct answer is (C).

If labeling the equation isn't enough to eliminate three answers, try plugging in! This will help clarify what's going on in confusing equations or show the relationship between two parts of an equation.

Let's try a meaning in context question that uses plugging in.

 Mark for Review

$$n = 1,273 - 4p$$

11. The equation above was used by the cafeteria in a large public high school to model the relationship between the number of slices of pizza, n , sold daily and the price of a slice of pizza, p , in dollars. What does the number 4 represent in this equation?

- ☐ A For every \$4 the price of pizza decreases, the cafeteria sells 1 more slice of pizza.
- ☐ B For every dollar the price of pizza decreases, the cafeteria sells 4 more slices of pizza.
- ☐ C For every \$4 the price of pizza increases, the cafeteria sells 1 more slice of pizza.
- ☐ D For every dollar the price of pizza increases, the cafeteria sells 4 more slices of pizza.

Here's How to Crack It

The question asks for the meaning of the number 4 in the context of the situation. First, read the question very carefully, and label the variables on your scratch paper. You know that p is the price of pizza, and n is the number of slices, so you can add that information to the equation. If you can, eliminate answer choices that don't make sense. But what if you can't eliminate anything, or you can eliminate only an answer choice or two?

Even with everything labeled, this equation is difficult to decode, so it's time to plug in! Try a few of your own numbers in the equation, and you will get a much better understanding of what is happening.

Try it out with $p = 2$. When you put 2 in for p , $n = 1,273 - 4(2)$ or 1,265.

So, when $p = 2$, $n = 1,265$. In other words, at \$2 a slice, the cafeteria sells 1,265 slices.

When $p = 3$, $n = 1,261$, so at \$3 a slice, the cafeteria sells 1,261 slices.

When $p = 4$, $n = 1,257$, so at \$4 a slice, the cafeteria sells 1,257 slices.

Now, use POE. First of all, is the cafeteria selling more pizza as the price goes up? No, as the price of pizza goes up, the cafeteria sells fewer slices of pizza. That means you can eliminate (C) and (D).

Choice (A) says that for every \$4 the price goes down, the cafeteria sells 1 more slice of pizza. Does your plugging in back that up? No. The cafeteria sells 8 more slices of pizza when the price drops from \$4 to \$2, so it will sell 16 more slices of pizza when the price drops by \$4, and (A) is no good.

Now take a look at (B). Does the cafeteria sell 4 more slices of pizza for every dollar the price drops? Yes! The correct answer is (B).



Here are the steps for solving meaning in context questions. The first three steps are often enough, but plug in when they aren't.

Meaning in Context



1. Read the question carefully. Make sure you know which part of the equation you are being asked to identify.
2. Use your scratch paper to rewrite the equation, replacing the parts of the equation you can identify with labels.

3. Eliminate any answer choices that clearly describe the wrong part of the equation, or go against what you have labeled.
4. Plug in! Use your own numbers to start seeing what is happening in the equation.
5. Use POE again, using the information you learned from plugging in real numbers, until you can get it down to one answer choice. Or get it down to as few choices as you can, and guess.

Let's look at a slightly different one now.

 Mark for Review

$$7x + y = 133$$

12. Jeffrey has set a monthly budget for purchasing frozen blended mocha drinks from his local coffee shop. The equation above can be used to model the amount of his budget, y , in dollars, that remains after buying coffee for x days in a month. What does it mean that $(19, 0)$ is a solution to this equation?

- ☐ A Jeffrey starts the month with a budget of \$19.
- ☐ B Jeffrey spends \$19 on coffee every day.
- ☐ C It takes 19 days for Jeffrey to drink 133 cups of coffee.
- ☐ D It takes 19 days for Jeffrey to run out of money in his budget for purchasing coffee.

Here's How to Crack It

The question asks about a point that is the solution to an equation in the context of the situation. Start by labeling the x and the y in the equation to keep track of what they stand for. Use your scratch paper to replace x with “days” and y with “budget.” So $7 \times \text{days} + \text{budget} = 133$. Hmm, still not very clear, is it? One way to approach this is to plug in the point. If $x = \text{days} = 19$ when $y = \text{budget} = 0$, then Jeffrey will have no budget left after 19 days. This matches (D).

If you have trouble seeing this, you can use the answer choices to help you plug in. If (A) is true, the budget at the start of the month, when $\text{days} = 0$, is \$19. Plug these values into the equation to see if it is true. Is $7 \times 0 + 19 = 133$? Not at all, so eliminate (A). If (B) is true, Jeffrey drinks a lot of coffee! Try some numbers and see if it works. For $x = 1$, the equation becomes $7(1) + y = 133$ or $y = 126$, and for $x = 2$, it is $7(2) + y = 133$ or $y = 119$. The difference in y , the budget remaining, is $126 - 119 = 7$, so that's not \$19 per day. Eliminate (B) so only (C) and (D) remain. These both have 19 for the number of days, and the point $(19, 0)$ would indicate that 19 is the x -value, or days. If you saw that right away—great! That would allow you to skip right to testing (C) and (D).

For (C), you can plug in 19 for days in the equation to get $7 \times 19 + \text{budget} = 133$, or $\text{budget} = 0$. Does that tell you how many cups of coffee Jeffrey drank? You have no information about the cost of a single cup of coffee, so the answer can't be (C). It does tell you, however, that after 19 days, Jeffrey has no budget left. The correct answer is (D).

You might have noticed that the equation in this question could have been graphed in the xy -plane, which could help you visualize the situation. Some Digital SAT questions ask about the meaning of a piece of a linear graph, so keep the meaning in context approach in mind when you get to the Functions and Graphs chapter.



Other Digital SAT Algebra Strategies Drill

Work these algebra questions using Plugging In or PITA. Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. The length of a certain rectangle is twice the width. If the area of the rectangle is 128, what is the length of the rectangle?

☐ A 4

☐ B 8

☐ C 16

☐ D $21\frac{1}{3}$

 Mark for Review

2. If $xy < 0$, which of the following must be true?

I. $x + y = 0$

II. $2y - 2x < 0$

III. $x^2 + y^2 > 0$

☐ A I only

☐ B III only

☐ C I and III

☐ D II and III

 Mark for Review

3. If $\frac{\sqrt{x}}{2} = 2\sqrt{2}$, what is the value of x ?

☐ A 4

☐ B 16

☐ C $16\sqrt{2}$

☐ D 32

 Mark for Review

$$\sqrt{2x - k} = 3 - x$$

4. If $k = 3$, which of the following contains all possible solutions to the equation above?

☐ A -2

☐ B 2

☐ C 2 and 6

☐ D 6

 Mark for Review

5. If Alex can fold 12 napkins in x minutes, how many napkins can he fold in y hours?

☐ A $\frac{720}{xy}$

☐ B $\frac{xy}{720}$

☐ C $\frac{720y}{x}$

☐ D $\frac{720x}{y}$

 Mark for Review

6. Nails are sold in 8-ounce and 20-ounce boxes. If 50 boxes of nails were sold and the total weight of the nails sold was less than 600 ounces, what is the greatest possible number of 20-ounce boxes that could have been sold?

☐ A 16

☐ B 17

☐ C 25

☐ D 33

 Mark for Review

7. In a certain school, 55% of the students sing in the choir. Of those who do not sing in the choir, 80% play in the band. If all students at the school participate in exactly one musical activity, what percentage of the students in the school participate in a musical activity other than singing in the choir or playing in the band?

☐ A 9%

☐ B 11%

☐ C 36%

☐ D 44%

 Mark for Review

8. The equation $ax^2 - 12x + 12 = 0$ has exactly one solution for which of the following values of constant a ?

☐ A -3

☐ B -2

☐ C 2

☐ D 3

 Mark for Review

9. A gas station sells regular gasoline for \$2.39 per gallon and premium gasoline for \$2.79 per gallon. If the gas station sold a total of 550 gallons of both types of gasoline in one day for a total of \$1,344.50, how many gallons of premium gasoline were sold?

☐ A 25

☐ B 75

☐ C 175

☐ D 475

 Mark for Review

10. On a test, all right answers are worth the same number of points and all wrong answers are worth the same number of points. The equation $25C - 20W = 400$ represents the number of right answers, C , and wrong answers, W , a student had on a test when the student earned a total of 400 points. The point value of a right answer is how much more than the point value of a wrong answer?

☐ A 20

☐ B 25

☐ C 45

☐ D 80

OTHER DIGITAL SAT ALGEBRA STRATEGIES DRILL ANSWERS AND EXPLANATIONS

1. C

The question asks for the length of the rectangle. This is a specific value, and there are numbers in the answer choices, so plug in the answers. If you start with (B), the length is 8, and the width is half that, or 4. Area is length $\times$ width. The area of this rectangle is 8×4 , which is nowhere near 128. Eliminate (A) and (B), as both are too small. Try (C). If the length is 16, the width is 8. So, does $128 = 16 \times 8$? Yes, it does! Since this matches the information in the question, stop here. The correct answer is (C).

2. B

The question asks which statements must be true. A question with unknown variables indicates a good place to plug in. You need

numbers for x and y that will give you a negative product. Try $x = 1$ and $y = -2$. If you plug these into the statements in the Roman numerals, you find that (I) is false, but (II) and (III) are true. You can eliminate any answer choice that contains (I). This leaves (B) and (D). Now try different numbers to see if you can eliminate another choice. If you try $x = -1$ and $y = 2$, you find that (II) is false and (III) is still true. The correct answer is (B).

3. **D**

The question asks for the value of x . This is a specific amount, and there are numbers in the answer choices, so plug in the answers, starting with (B). If $x = 16$, the left side of the equation is $\frac{\sqrt{16}}{2} = \frac{4}{2} = 2$. Does that equal $2\sqrt{2}$? No—it's too small. Choice (C) seems tough to work with, so try (D) next. If it is too big, (C) is your answer. For (D), $x = 32$, and the left side of the equation becomes $\frac{\sqrt{32}}{2} = \frac{\sqrt{16 \times 2}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$. It's a match. The correct answer is (D).

4. **B**

The question asks for the possible solutions to an equation. Since the question asks for a specific value and the answers contain numbers in increasing order, plug in the answers. Write the answers on the scratch paper, label them as " x ," and start with a number that appears more than once in the answers, such as 2. Plug $x = 2$ and $k = 3$ into the

equation to get $\sqrt{2(2)-3} = 3-2$, which becomes $\sqrt{4-3} = 1$, then $\sqrt{1} = 1$, and finally $1 = 1$. This is true, so eliminate (A) and (D) because they do not include 2. Try 6 next, and plug $x = 6$ and $k = 3$ into the equation to get $\sqrt{2(6)-3} = 3-6$, which becomes $\sqrt{12-3} = -3$, then $\sqrt{9} = -3$, and finally $3 = -3$. (On the Digital SAT, the square root of a number is always positive.) This is not true, so eliminate (C). The correct answer is (B).

5. C

The question asks for the number of napkins that can be folded in y hours. The two variables tell you this is a great place to plug in. Pick numbers that make the math easy. You can try $x = 30$ and $y = 2$. So in 2 hours there are 4 periods of 30 minutes each: $12 \times 4 = 48$. Alex can fold 48 napkins in 2 hours, so 48 is your target. Plug the values for x and y into the answer choices to see which one matches the target. Only (C) works, so the correct answer is (C).

6. A

The question asks for the greatest possible number of 20-ounce boxes. Because this is a specific amount and there are numbers in the answer choices, this is a perfect opportunity to use PITA. Start with (C). If there are twenty-five 20-ounce boxes, then there are twenty-five 8-ounce boxes because a total of 50 boxes was purchased. In this case, the twenty-five 20-ounce boxes weigh 500 ounces, and the twenty-five 8-ounce boxes weigh 200 ounces; the total is 700 ounces. This is too big because the question says the total weight was less than 600. If (C) is too big, (D) must also be too big; eliminate both answers. If you

try (B), the total weight is 604 ounces, which is still too big. Thus, the correct answer is (A).

7. **A**

The question asks for a percentage. No specific values are given, so plug in. Since *percent* means “out of 100,” plug in 100 for the total number of students to make the math easy. Next, work the steps of the question one at a time. The question states that *55% of the students sing in the choir*, so 55 of the 100 students sing in the choir. Read carefully: the next step refers to *those who do not sing in the choir*. If 55 sing in the choir, $100 - 55 = 45$ do not sing in the choir. Take 80%, or $\frac{80}{100}$, of those 45 students to get $\frac{80}{100}(45) = 36$ students who play in the band. That leaves $100 - 55 - 36 = 9$ students who participate in a musical activity other than choir or band. As a result of plugging in 100, there is no need to do math to convert to a percentage: 9 out of 100 is 9%. The correct answer is (A).

8. **D**

The question asks for the value of a constant in a quadratic equation. One way to solve the question is to work with the discriminant, which you learned about in Chapter 21. However, the built-in calculator or a personal graphing calculator, combined with plugging in the answers, will get the question right faster. Start with (C), and plug in 2 for a . The equation becomes $2x^2 - 12x + 12 = 0$. Enter the left side of the equation into the built-in calculator. There are two dots where the

graph intersects the x -axis, which means the quadratic has two solutions. Eliminate (C). It might not be obvious whether a bigger or smaller number is needed, but it won't take long to try other numbers. Try (B) and replace 2 with -2 in the equation. The calculator shows that the quadratic again has two solutions; eliminate (B). Try (D) and replace -2 with 3. The quadratic now has one solution at $(2, 0)$. The correct answer is (D).

9. **B**

The question asks for the number of gallons of premium gasoline that were sold. When asked for a specific value, try plugging in the answers. Label them as gallons of premium and start with the value in (B). If 75 gallons of premium were sold, the station would make $75(\$2.79) = \209.25 for those sales. A total of 550 gallons was sold, so the station would have sold $550 - 75 = 475$ gallons of regular gasoline. The sales for the regular gasoline would be $475(\$2.39) = \$1,135.25$. The total sales for both types of gasoline would be $\$209.25 + \$1,135.25 = \$1,344.50$. That matches the information in the question, so the correct answer is (B).

10. **C**

The question asks for the relationship between two values in an equation. It will help to understand what the parts of the equation represent, so label parts of the equation and rewrite it. The question states that C represents the number of right answers, W represents the number of wrong answers, and 400 represents the total number of points. Rewrite the equation as $25(\text{number of right answers}) - 20(\text{number of wrong answers}) = (\text{total number of points})$. Thus, 25 and -20 must represent the number of points earned for each right and wrong answer, respectively. To determine how much more a right answer is worth than a wrong answer, subtract the values to get $25 - (-20) = 45$. The correct answer is (C).

Summary

- When an algebra question asks for a specific amount and has numbers in the answer choices, plug each of the answer choices into the problem until you find one that works.
- If you start with one of the middle numbers, you may be able to cut your work. The answer choices will be in order, so if your number is too high or too low, you'll know what to eliminate.
- When the question has variables in the answer choices, you can often plug in your own amounts for the unknowns and do arithmetic instead of algebra.
- When you plug in, use “good” numbers—ones that are simple to work with and that make the problem easier to manipulate: 2, 5, 10, or 100 are generally easy numbers to use.
- On meaning in context questions, label the parts of the equation with information from the question, and eliminate answers that don't match the labels.
- Plugging In can also be used on meaning in context questions. If labeling the answers and using Process of Elimination isn't enough to answer the question, plug your own amounts into the equation so you can start to see what is going on.

Chapter 23

Functions and Graphs

The Digital SAT includes a large number of questions about functions, either by themselves or connected to a graph. Some of these questions can get quite complicated, but the same basic concepts apply no matter what. This chapter will give you the tools you need to work on questions about functions and understand how functions and graphs are connected.

CALL ON THE CALCULATOR

A complete understanding of a graphing calculator can make a significant difference in how long it takes to answer questions about the graphs of functions. Try solving the examples with the built-in calculator or your own to discover how powerful it can be. Some questions cannot be solved by the graphing calculator, however, so it's also important to understand the concepts covered in this chapter.

FUNCTION FUNDAMENTALS

Think of a **function** as just a machine for producing ordered pairs. You put in one number and the machine spits out another. Think of these as the input and the output. An input value, x , gets put into the function machine, f , and an output value, y , comes out of the function machine. The most common function is an $f(x)$ function. A simple way to keep functions straight is to remember that $f(x) = y$.

Let's look at a question.

 Mark for Review

1. If $f(x) = x^3 - 4x + 8$, what is the value of $f(5)$?

☐ A 67

☐ B 97

☐ C 113

☐ D 147

What's This?

Anytime you see the notation $f(x)$, know that f isn't a variable; it's the name of the function. When you say it out loud, it's "f of x." Though $f(x)$ is the most common way to show that an equation is a mathematic function, any letter can be used. So you may see $g(x)$ or $h(d)$. Know that you're still dealing with a function.

Here's How to Crack It

The question asks for the value of $f(5)$ for the given function. Any time you see a number inside the parentheses, such as $f(5)$, plug in that number for x . The question is actually telling you to use Plugging In! Let's do it:

$$f(5) = 5^3 - 4(5) + 8$$

$$f(5) = 125 - 20 + 8$$

$$f(5) = 113$$

The correct answer is (C).



The previous question gave you a number to put into the function, which made it a Plugging In question. If the question gives you information about what comes out of the function and asks what should go in, it's a PITA question!

Here's an example of using PITA on a function question.



Mark for Review

2. For what value of x does $f(x) = 12$ if the function f is defined as $f(x) = \frac{1}{2}x + 2$?

☐ A 5

☐ B 8

☐ C 20

☐ D 24



Use PITA!

Don't forget that you can often plug in the answer choices on function questions! Noticing a pattern yet? Just a few easy tricks can unlock a lot of easy points.

Here's How to Crack It

The question asks for the input value of a function that results in a specific output value. The answer choices contain numbers in increasing order, and one of them must be the input that will result in an output of 12. Plug in the answers. Rewrite the answer choices on the scratch paper and label them “ x .” Next, start with one of the middle numbers. Try (B), 8. Plug $x = 8$ into the function to get $f(8) = \frac{1}{2}(8) + 2$, or $f(8) = 4 + 2$, and finally $f(8) = 6$. This does not match the output value of 12, so eliminate (B). The result was too

small, so try a larger number and plug in $x = 20$. The function becomes $f(20) = \frac{1}{2}(20) + 2$, or $f(20) = 10 + 2$, and finally $f(20) = 12$. This matches the output value of 12, so stop here. The correct answer is (C).

In this case, it would have been possible to set the function equal to 12 and solve for x . However, solving will be difficult on more complicated problems, and it's easy to make mistakes with algebra. PITA is always a good idea when a question gives the output of a function and asks for the input.



Now that you know the basics of functions on the Digital SAT, try the next one.



 Mark for Review

3. If $f(x) = x^2 + 2$, which of the following could be a value of $f(x)$?

☐ A -1

☐ B 0

☐ C 1

☐ D 2

Here's How to Crack It

The question asks for a possible value of the function. Therefore, the question is asking which of these values could be spit out of the $f(x)$ machine. Think about what is going in. No matter what you put in as a

value for x , the value of x^2 has to be positive or zero. So, the lowest possible value of $x^2 + 2$ is 2. The correct answer is (D).

Note that you could also approach this question by plugging in the answers. If you plugged in 1 for $f(x)$, for instance, you would get $1 = x^2 + 2$, which becomes $x^2 = -1$, which is impossible.



Sometimes you'll get more complicated questions. As long as you know that when you put in x , your function will spit out another number, you'll be fine. Now try another one.



 Mark for Review

4. What is the value of c if $g(x) = 4x^2 - 6$ and $g(x - c) = 4x^2 + 40x + 94$?

☐ A -40

☐ B -5

☐ C 5

☐ D 40

Here's How to Crack It

The question asks for the value of c . In function notation, the number inside the parentheses is the x -value that goes into the function. This question might seem complicated, but look beyond the math and use your Princeton Review Digital SAT knowledge. The question asks for a specific value and has numbers in the answers, so try PITA! Label the answer choices as “ c ”

and start in the middle with (C), 5. Plug $c = 5$ into the question, and the value to put into the function becomes $x - 5$.

$$g(x - 5) = 4(x - 5)^2 - 6$$

FOIL the quadratic to get

$$g(x - 5) = 4(x^2 - 10x + 25) - 6$$

Distribute and combine like terms to get

$$\begin{aligned} g(x - 5) &= 4x^2 - 40x + 100 - 6 \\ g(x - 5) &= 4x^2 - 40x + 94 \end{aligned}$$

Set the results equal to the quadratic given in the question.

$$4x^2 - 40x + 94 = 4x^2 + 40x + 94$$

This is not true, so eliminate (C). The middle term has the right coefficient but the wrong sign, so try (B), -5 . The function becomes

$$g(x + 5) = 4(x + 5)^2 - 6$$

Carry out the same steps as before to get

$$\begin{aligned} g(x + 5) &= 4(x^2 + 10x + 25) - 6 \\ g(x + 5) &= 4x^2 + 40x + 100 - 6 \\ g(x + 5) &= 4x^2 + 40x + 94 \end{aligned}$$

This is equal to the quadratic given in the question, so -5 is the value of c . The correct answer is (B).



Sometimes you may see a word problem that describes a function and then asks you to “build a function” that describes the real-world situation presented in the question. Take the following question, for example.

 Mark for Review

5. Rock climbing routes are rated on a numbered scale with the highest number representing the most difficult route. Sally tried a range of shoe sizes on each of several routes of varying difficulty and found that when she wore smaller shoes, she could climb routes of greater difficulty. If D represents the difficulty rating of a route Sally successfully climbed and s represents the size of the shoes she wore on such a route, then which of the following could express D as a function of s ?

☐ A $D(s) = s^2$

☐ B $D(s) = \sqrt{s}$

☐ C $D(s) = s - 3.5$

☐ D $D(s) = \frac{45}{s}$

Here's How to Crack It

The question asks for a function that best represents a situation. Start by thinking about the relationship described in the question: the smaller the shoes, the greater the difficulty. This is an inverse relationship. So, look for an inverse function. Only (D) expresses this type of relationship because a greater value of s leads to a smaller value of D .

If you aren't sure, try plugging in numbers to try it out. Plug in $s = 8$ and then $s = 10$ to see if the result for D is smaller when you use a larger shoe size. Since only (D) results in a smaller difficulty for a larger shoe size, the correct answer is (D).

One way the test-writers will make functions more complicated is to use percentages. Use your knowledge of function basics from this chapter and of percentages from earlier in this book to work through these questions.

Try this example.

 Mark for Review

6. For every increase in the value of x by 1 in function z , the value of $z(x)$ increases by 25%. Which of the following functions defines z if $z(1) = 12.5$?

☐ A $z(x) = 10(1.25)^x$

☐ B $z(x) = 1.25(10)^x$

☐ C $z(x) = 0.75(10)^x$

☐ D $z(x) = 10(0.75)^x$

Here's How to Crack It

The question asks for the equation that defines a function. Recall that, in function notation, the number inside the parentheses is the x -value that goes into the function, or the input, and the value that comes out of the function is the y -value, or the output. The question provides a pair of input and

output values by stating that $z(1) = 12.5$. Plug $x = 1$ into the answer choices and eliminate answers that do not result in $z(1) = 12.5$. These are the results:

A)

$$z(1) = 10(1.25)^1$$

$$z(1) = 10(1.25)$$

$$z(1) = 12.5 \text{ Keep (A).}$$

B)

$$z(1) = 1.25(10)^1$$

$$z(1) = 1.25(10)$$

$$z(1) = 12.5 \text{ Keep (B).}$$

C)

$$z(1) = 0.75(10)^1$$

$$z(1) = 0.75(10)$$

$$z(1) = 7.5 \text{ Eliminate (C).}$$

D)

$$z(1) = 10(0.75)^1$$

$$z(1) = 10(0.75)$$

$$z(1) = 7.5 \text{ Eliminate (D).}$$

Two answers worked, so try a different value. The question states that *for every increase in the value of x by 1 in function z , the value of $z(x)$ increases by 25%*. Increase the input value by 1 to get $x = 1 + 1 = 2$. When $x = 1$, $z(x) = 12.5$, so when $x = 2$, $z(x)$ will increase by 25%. Take the value of $z(1)$, which is 12.5, and add 25% of it. *Percent* means out of 100, so translate 25% as $\frac{25}{100}$. Thus, $z(2) = 12.5 + \left(\frac{25}{100}\right)(12.5)$, or $z(2) = 15.625$. The other option is to use the built-in calculator, which automatically translates the % symbol and adds the word “of” to multiply the percent by the next number

you enter. Either way, the target value for $z(2)$ is 15.625. Plug $x = 2$ into the remaining answer choices:

A)

$$z(2) = 10(1.25)^2$$

$$z(2) = 10(1.5625)$$

$$z(2) = 15.625 \text{ Keep (A).}$$

B)

$$z(2) = 1.25(10)^2$$

$$z(2) = 1.25(100)$$

$$z(2) = 125 \text{ Eliminate (B).}$$

Only one answer remains, so it must be correct. If the functions in the answer choices look familiar, that's because they are in the form of the growth and decay formula you learned about in Chapter 21: *final amount* = *original amount* $(1 \pm \text{rate})^{\text{number of changes}}$. In correct answer (A), $z(x)$ is the *final amount*, 10 is the *original amount*, 0.25 is the *rate*, and x is the number of changes. But don't worry if you didn't catch that: plugging in values from the question was all you needed to do to get this one right. The correct answer is (A).

Functions and Tables

There is one more way the test-writers will test you on input and output values in a function, and that's by putting the values into a table. Stick to the basics of functions and look for chances to plug in.

Take a look at the following example.

 Mark for Review

x	$f(x)$
1	8
2	17
3	26
4	35

7. Linear function f is represented in the table above, which shows four values of x and their corresponding values of $f(x)$. Which of the following functions accurately represents function f ?

☐ A) $f(x) = 8x$

☐ B) $f(x) = 8x + 8$

☐ C) $f(x) = 9x - 1$

☐ D) $f(x) = 9x + 8$

Here's How to Crack It

The question asks for the function that represents values given in a table. Apply the function basics: x is the input, and $f(x)$, which is equivalent to y , is the output of the function. The table shows pairs of values for x and $f(x)$, and the correct function must work for every pair of values. Plug in values from the table and eliminate functions that don't work. Plug in $x = 1$ and $f(x) = 8$ to get the following results:

A) $8 = 8(1)$, or $8 = 8$

True, so keep (A).

B) $8 = 8(1) + 8$, or $8 = 16$

Not true; eliminate (B).

C) $8 = 9(1) - 1$, or $8 = 8$

True, so keep (C).

D) $8 = 9(1) + 8$, or $8 = 17$

Not true; eliminate (D).

Two answers worked for the first pair of input and output values, so try a second pair from the table. Plug $x = 2$ and $y = 17$ into the remaining answers to get these results:

A) $17 = 8(2)$, or $17 = 16$

Not true; eliminate (A).

C) $17 = 9(2) - 1$, or $17 = 17$

True, so keep (C).

The only answer that worked both times must be correct, so don't spend time trying the other two points. The correct answer is (C).

What's the Point?

Why did math folks come up with functions? To graph them, of course! When you put in a value for x , and your machine (or function) spits out another number, that's your y . You now have an ordered pair. Functions are just another way to express graphs. Knowing the connection between functions and graphs is useful, because you will most likely see questions involving graphs on the Digital SAT.

THE COORDINATE PLANE

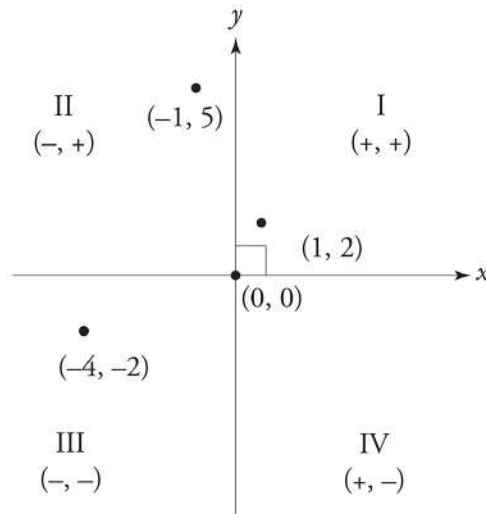
A **coordinate plane**, or the **xy-plane**, is made up of two number lines that intersect at a right angle. The horizontal number line is called the **x-axis**, and the vertical number line is the **y-axis**.

The four areas formed by the intersection of the axes are called **quadrants**. The location of any point can be described with a pair of numbers (x, y), just the way you would describe a point on a map: $(0, 0)$ are the coordinates of the intersection of the two axes (also called the **origin**); $(1, 2)$ are the coordinates of the point one space to the right and two spaces up; $(-1, 5)$ are

the coordinates of the point one space to the left and five spaces up; $(-4, -2)$ are the coordinates of the point four spaces to the left and two spaces down. All of these points are located on the following figure.

Quadrants

A coordinate plane has four distinct areas known as quadrants. The quadrants are numbered counterclockwise, starting from the top right. They help determine generally whether x and y are positive or negative. Sometimes knowing what quadrant a point is in and what that means is all you need to find the answer.



Some of the questions on the Digital SAT may require you to know certain properties of lines on the xy -plane. Let's talk about them.

Ways to Remember

Having trouble remembering that the x -coordinate comes before the y -coordinate in an ordered pair? Just remember the phrase "x before y, walk before you fly." The letter x also comes before y in the dictionary.

POINTS ON A LINE

You may be asked if a point is on a line or on the graph of any other equation. Just plug the coordinates of the point into the equation of the line to determine if that point makes the equation a true statement.

 Mark for Review

8. In the xy -plane, which of the following ordered pairs is a point on the line $y = 2x - 6$?

☐ A (6, 7)

☐ B (7, 7)

☐ C (7, 8)

☐ D (8, 7)

Here's How to Crack It

The question asks for a point that is on the given line. Plug in the answers, starting with (B). The (x, y) point is $(7, 7)$, so plug in 7 for x and 7 for y . The equation becomes $7 = 2(7) - 6$ or $7 = 8$. This isn't true, so eliminate (B). The result was very close to a true statement, and the point in (C) has the same x -coordinate and a larger y -coordinate, so try that next. Because $8 = 2(7) - 6$, the correct answer is (C).

SLOPE

You always read a graph from left to right. As you read the graph, how much the line goes up or down is known as the slope. **Slope** is the rate of

change of a line and is commonly known as “rise over run.” It’s denoted by the letter m . Essentially, it’s the change in the y -coordinates over the change in the x -coordinates and can be found with the following formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

This formula uses the points (x_1, y_1) and (x_2, y_2) . For example, if you have the points $(2, 3)$ and $(7, 4)$, the slope of the line created by these points would be

$$m = \frac{(4 - 3)}{(7 - 2)}$$

So the slope of a line with points $(2, 3)$ and $(7, 4)$ would be $\frac{1}{5}$, which means that every time you go up 1 unit, you travel to the right 5 units.

EQUATIONS OF A LINE

Slope-Intercept Form

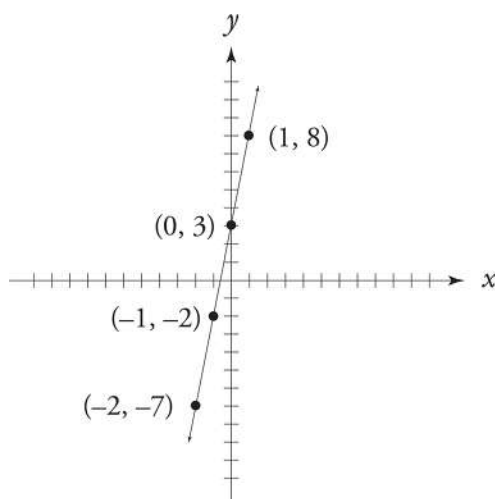
The equation of a line can take multiple forms. The most common of these is known as the **slope-intercept form**. If you know the slope and the y -intercept, you can create the equation of a given line. A slope-intercept equation takes the form $y = mx + b$, where m is the slope and b is the **y -intercept** (the point where the function crosses the y -axis).

Let’s say that you know that a certain line has a slope of 5 (which is the same as $\frac{5}{1}$) and a y -intercept of 3. The equation of the line would be $y = 5x + 3$. You could graph this line simply by looking at this

form of the equation. First, draw the y -intercept, $(0, 3)$. Next, plug in a number for x and solve for y to get a coordinate pair of a point on the line. Then connect the point you just found with the y -intercept you already drew, and voilà, you have a line. If you want more points, you can create a table such as the following:

x	y
-2	-7
-1	-2
0	3
1	8

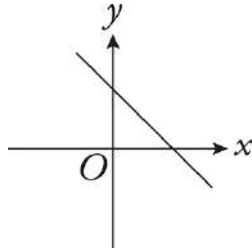
Take a look at the finished product:



One way the Digital SAT can test your understanding of lines is to show you a graph and ask you which equation describes that graph.

Here's an example.

 Mark for Review



9. Which of the following could be the equation of the line represented in the graph above?

☐ A $y = -2x - 1$

☐ B $y = -2x + 4$

☐ C $y = 2x - 4$

☐ D $y = 2x + 4$

Here's How to Crack It

The question asks for the equation of a line based on the graph. Remember that the equation of a line is $y = mx + b$, where m is the slope and b is the y -intercept. Look at the graph and think about what the equation should look like. Since the line is sloping downward, it should have a negative slope, so you can eliminate (C) and (D). Next, since the line has a positive y -intercept, you can eliminate (A). The correct answer is (B).

Standard Form

Another way the equation of a line can be written is the **standard form** of $Ax + By = C$, where A , B , and C are constants and A and B do not equal

zero. The test-writers will sometimes present equations in this form in the hopes that you will waste time putting it in slope-intercept form. If you know what to look for, the standard form can be just as useful as the slope-intercept form.

In standard form $Ax + By = C$:

- the slope of the line is $-\frac{A}{B}$
- the y-intercept of the line is $\frac{C}{B}$
- the x-intercept of the line is $\frac{C}{A}$

The equation $y = 5x + 3$ in the previous example would be $-5x + y = 3$ when written in the standard form. Using the information above, you can see that:

$$\text{slope} = -\left(\frac{-5}{1}\right) = 5$$

$$\text{y-intercept} = \frac{3}{1} = 3$$

$$\text{x-intercept} = \frac{3}{-5} = -\frac{3}{5}$$

The answers for the slope and the y-intercept were the same as when the slope-intercept form was used. Depending on the form of the equation in the question or in the answers, knowing these line equation facts can help save time on the test.

Let's look at how this may be tested.

 Mark for Review

10. The graph of which of the following equations is parallel to the line with equation $y = -\frac{1}{3}x - \frac{1}{6}$?

☐ A $x - \frac{1}{3}y = 3$

☐ B $x - 3y = 2$

☐ C $x + 6y = 4$

☐ D $x + 3y = 5$

Here's How to Crack It

The question asks for the equation of a line that has a slope parallel to the slope of the line given. In the form $y = mx + b$, m represents the slope. The equation in the question is in that form, so the slope is $-\frac{1}{3}$. Parallel lines have equal slopes, so all you need to do now is find the answer choice that also has a slope of $-\frac{1}{3}$.

One way to do that would be to rewrite each answer in the $y = mx + b$ form. However, notice that the equations in the answer choices are in the $Ax + By = C$ form, and in that form the slope is equal to $-\frac{A}{B}$. Find the slope of each answer choice, and eliminate the ones that are not $-\frac{1}{3}$. The slope of the line

in (A) is $-\frac{1}{-\frac{1}{3}} = 3$. This is not the correct slope, so eliminate (A). The slope of the line in (B) is $-\frac{1}{-3} = \frac{1}{3}$. This is also the wrong slope, so eliminate (B). The slope of the line in (C) is $-\frac{1}{6}$, which is also the wrong slope, so eliminate (C). The slope of the line in (D) is $-\frac{1}{3}$. This is the same slope as the line given in the question. The correct answer is (D).



PARALLEL AND PERPENDICULAR LINES

So now you know that **parallel lines** have the same slope. Whenever the Digital SAT brings up **perpendicular lines**, just remember that a perpendicular line has a slope that is the *negative reciprocal* of the other line's slope. For instance, if the slope of a line is 3, then the slope of a line perpendicular to it would be $-\frac{1}{3}$. Combine this with the skills you've already learned to work on a question about perpendicular lines.

Parallel vs. Perpendicular

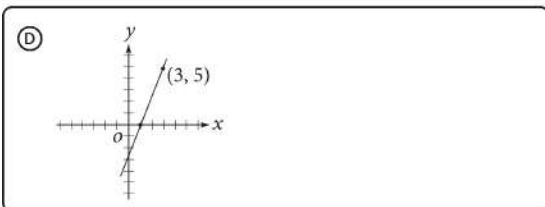
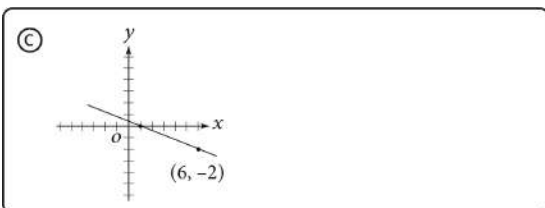
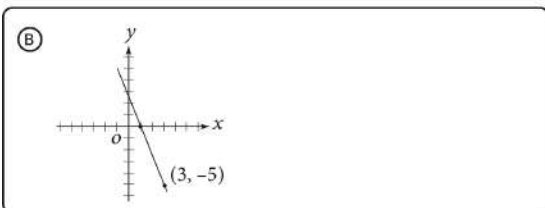
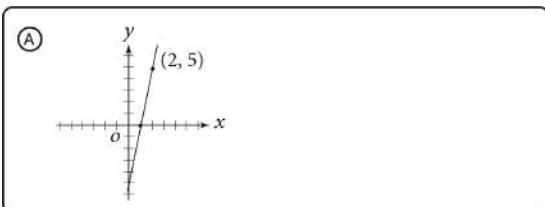
Parallel lines have the same slope and never intersect. Perpendicular lines have slopes that are negative reciprocals and intersect at a right angle.

Here's an example.



 Mark for Review

11. Which of the following is the graph of a line perpendicular to the line defined by the equation $2x + 5y = 10$?



Here's How to Crack It

The question asks for a line perpendicular to the line $2x + 5y = 10$. Therefore, you need to find the slope of the line and then take the negative reciprocal to find the slope of the perpendicular line. You can convert the equation into the $y = mx + b$ format in order to find the slope, or simply remember that when an equation is presented in the form $Ax + By = C$, the

slope is equal to $-\frac{A}{B}$. So the slope of this line is $-\frac{2}{5}$, and the slope of a perpendicular line would be $\frac{5}{2}$.

Look at the answer choices for one with a positive (upward) slope. Choices (B) and (C) slope downward, so eliminate them. Next, use points in the graph to find the slope of each answer. Eliminate (A); it has points at (1, 0) and (2, 5), for a slope of 5—too steep. The only remaining choice is (D), so the correct answer is (D).

TWO EQUATIONS WITH INFINITELY MANY SOLUTIONS

In the previous chapters on algebra, we discussed equations with one or multiple solutions. Now imagine an equation in which any value of x would create a viable solution to the equation.

$$x + 3 = x + 3$$

In this case, it is fairly obvious that any number you choose to put in for x will create a true equation. But what does it mean when two lines have infinitely many solutions? Let's look at an example.

To Infinity...and Beyond!

When given two equations with infinitely many solutions, find a way to make them equal. The equations represent the same line.

 Mark for Review

$$gx - hy = 78$$

$$4x + 3y = 13$$

12. In the system of equations above, g and h are constants. If the system has infinitely many solutions, what is the value of gh ?

☐ A -432

☐ B -6

☐ C 6

☐ D 432

Here's How to Crack It

The question asks for the value of gh , where g and h are coefficients in the system of equations. This question may have you scratching your head and moving on to the next question, but explore what you can do to solve this before you decide it's not worth your time. You may be surprised by how easy it is to solve a problem like this.

When they say that these equations have infinitely many solutions, what they are really saying is that these are the same equation, or that one equation is a multiple of the other equation. In other words, these two equations represent the same line. With that in mind, try to determine what needs to be done to make these equations equal. Since the right side of the equation is dealing with only a constant, first determine what you would need to do to make 13 equal to 78.

In this case, you need to multiply 13 by 6. Since you are working with equations, you need to do the same thing to both sides of the equation in order for the equation to remain equal.

$$6(4x + 3y) = 6 \times 13$$

$$24x + 18y = 78$$

Since both equations are now equal to 78, you can set them equal to each other, giving you this equation:

$$24x + 18y = gx - hy$$

You may know that when you have equations with the same variables on each side, the coefficients on those variables must be equal, so you can deduce that $g = 24$ and $h = -18$. (Be cautious when you evaluate this equation. The test-writers are being sneaky by using addition in one equation and subtraction in another.) Therefore, gh equals $24 \times -18 = -432$. The correct answer is (A).

TWO EQUATIONS WITH NO SOLUTIONS

You saw above that a system of equations can have infinitely many solutions. When solving equations, you likely assume, as most people do, that there will be at least one solution to the equation, but that is not always the case. Look at the following example.

$$3x - 6 = 3x + 7$$

If you solve this equation, you will find that $-6 = 7$. Since -6 can never equal 7, there is no value of x that can be put into this equation to make it true. In this case, the equation has no solutions.

What does it mean if two equations of lines have no solutions? Here's one to try.

13. Which of the following accurately represents the set of solutions for the lines $6x + 12y = -24$ and $y = -\frac{1}{2}x + 2$?

☐ (A) $(0, -4)$

☐ (B) $(0, 4)$

☐ (C) There are no solutions.

☐ (D) There are infinitely many solutions.

There's Just No Solution

When given two equations with no solutions, find a way to compare slopes. The equations represent parallel lines.

Here's How to Crack It

The question asks for the solution to the system of equations. If the lines intersect, this will be the point of intersection. The answers in (C) and (D), though, suggest that the lines may be the same or parallel. Rather than plugging in the points in (A) and (B), look for a way to compare slopes. Start by putting the first line into $y = mx + b$ form: $12y = -6x - 24$. Divide the whole equation by 12, so $y = -\frac{1}{2}x - 2$. Since these lines have the same slope but different y -intercepts, the lines are parallel, and they will never intersect. Therefore, the correct answer is (C).

If two lines had different slopes, the lines would intersect at a single point such as (A) or (B). If the equations were identical, then they would be the

same line and therefore have infinitely many solutions.

POINTS OF INTERSECTION

Earlier in this book you learned how to find the solution to a system of equations. There are several ways to do this, including stacking up the equations and adding or subtracting, setting them equal, or even plugging in the answers. The Digital SAT may also ask about the intersection of the graphs of two equations in the xy -plane, which is a similar idea.

Let's try one.

 Mark for Review

14. In the xy -plane, which of the following is a point of intersection between the graphs of $y = x + 2$ and $y = x^2 + x - 2$?

☐ A (0, -2)

☐ B (0, 2)

☐ C (1, 0)

☐ D (2, 4)

Here's How to Crack It

The question asks for the point of intersection for two equations. This is a point that is on the graphs of both equations. Therefore, the point would work if plugged into the equation of the line and the equation of the parabola.



Here's how you would apply PITA in a point of intersection question.

So, use PITA by testing the answer choices: start with one of the answers in the middle and plug the point into each equation to see if it is true. The correct point of intersection will work in both functions. Try (C) in the first equation: does $0 = (1) + 2$? No. So, (C) is not the answer. Try (D) in the first equation: does $4 = (2) + 2$? Yes. So, try (D) in the second equation: does $4 = (2)2 + 2 - 2$? Yes. Since (2, 4) works in both equations, the correct answer is (D).

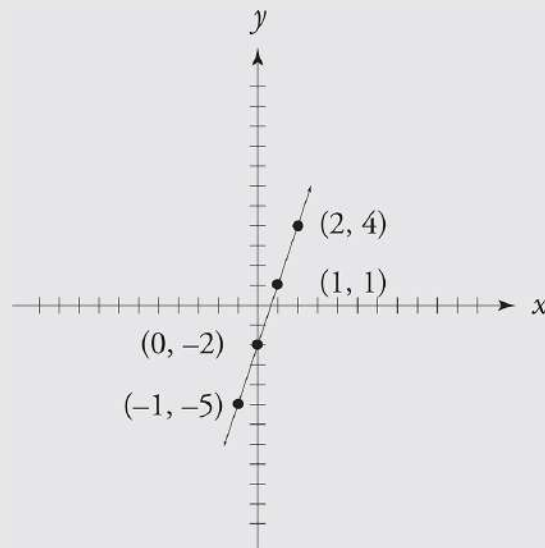
CALL ON THE CALCULATOR

It's good to know the concepts and math behind the graphs of lines and the way they're tested on the Digital SAT. It's also good to know how to use the built-in graphing calculator or your own that you bring with you on test day. Go back to some of the previous examples and try them again using a graphing calculator (if you didn't use it the first time). You might be surprised by how easy some of them are to answer.

$y = f(x)$

Sometimes, instead of seeing the typical $y = mx + b$ equation, or something similar, you'll see $f(x) = mx + b$. Look familiar? Graphs are just another way to show information from a function. Functions show information algebraically and graphs show functions geometrically (as pictured).

Here's an example. The function $f(x) = 3x - 2$ is shown graphically as the following:



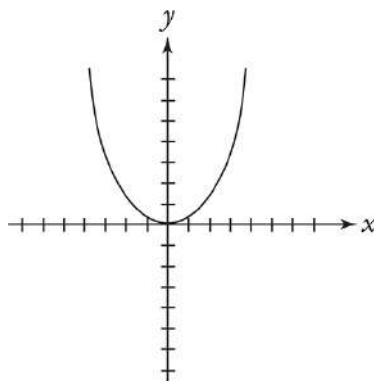
The reason the Digital SAT includes function questions is to test whether you can figure out the relationship between a function and its graph. To tackle these questions, you need to know that the independent variable, the x , is on the x -axis, and the dependent variable, the $f(x)$, is on the y -axis. For example, if you see a function of $f(x) = 7$, then you need to understand that this is a graph of a horizontal line where $y = 7$.

The rest of this chapter will deal with graphs of parabolas and circles instead of straight lines. The graphing calculator is extremely useful on many questions like this, and we'll show you how to take advantage of it.

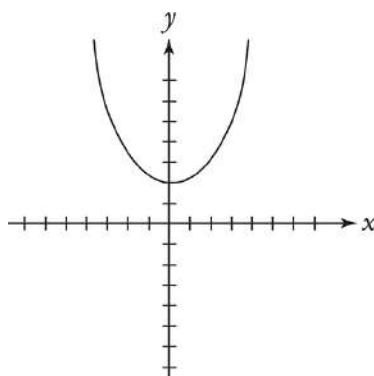
GRAPHING FUNCTIONS

One type of function question you might be asked is how the graph of a function would shift if you added a value to it.

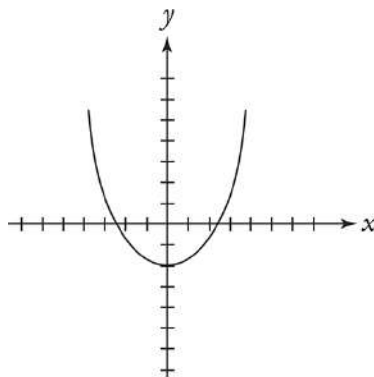
Here is a quick guide for the graph of $f(x) = x^2$, as seen below:



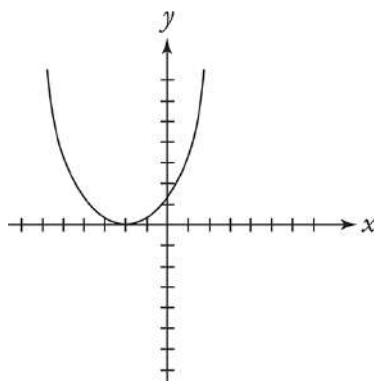
For $f(x) + c$, where c is a constant, the graph will shift up c units, as shown in the figure below:



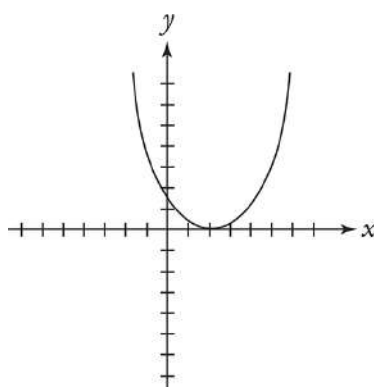
Conversely, $f(x) - c$ will shift the graph down by c units:



For $f(x + c)$, the graph will shift c units to the left:



For $f(x - c)$, the graph will shift to the right by c units:



You may have realized how easy these questions would become if you simply put them into your graphing calculator.

You can also plug in points to find the correct graph.

You have several options for dealing with questions like this. You can know the four rules for transforming graphs, you can plug in points to find the correct graph, or you can use a graphing calculator. The built-in calculator can graph the equation even if the test-writers give you an equation in an unusual form.

Try the next one using the built-in graphing calculator or your own.



 Mark for Review

15. Function f can be graphed in the xy -plane by translating function g down 3 units. Function g is defined by $g(x) = -2x^3 + 4x^2 - x + 2$. What is the value of $f(1)$?



Watch Us Crack It

Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for the value of a function. Start by graphing the equation of $g(x)$. In the built-in calculator, enter $g(x) = -2x^3 + 4x^2 - x + 2$ into the first entry field and you will see the graph in the graph display. The question asks about $f(1)$, so find the point where $x = 1$ on the graph of $g(x)$. You might need to scroll and zoom to see 1 on the x -axis, and use the mouse pointer to find the y -value when $x = 1$. The point is at $(1, 3)$. The question states that the graph of function f is the graph of function g translated down 3 units. Count 3 units down from the point at $(1, 3)$ in the graph of $g(x)$ to get to a point at $(1, 0)$. At this point, $x = 1$ and $y = 0$. Since $f(x) = y$, the value of $f(1)$ is 0. The correct answer is 0.



EQUATIONS OF A PARABOLA

Standard Form

The Digital SAT will ask questions using three different forms of the equation for a parabola.

The standard form of a parabola equation is

$$y = ax^2 + bx + c$$

In the standard form of a parabola, the value of a tells whether a parabola opens upward or downward (if a is positive, the parabola opens upward, and if a is negative, the parabola opens downward).

Factored Form

We looked at equations for parabolas in Chapter 21 when we solved quadratics. The factored form of a quadratic equation reveals the roots of the parabola. These are also the solutions of x . Given a question about roots or solutions, it can be helpful to know the relationship between the equation and the graph of the parabola in the xy -plane.

The factored form of a parabola equation is

$$y = a(x - r_1)(x - r_2)$$

In the factored form, r_1 and r_2 are the roots or x -intercepts of the parabola.

For the next question, the equation is $y = x^2 - 4x - 12$. If you factored this, you'd get $y = (x + 2)(x - 6)$, and the roots would be at $x = -2$ and $x = 6$. You can see that those are the exact points on the graph that the parabola crosses the x -axis.

Vertex Form

The vertex form of a parabola equation is

$$y = a(x - h)^2 + k$$

In the vertex form, the point (h, k) is the vertex of the parabola.

In vertex form, the value of a still indicates which way the parabola opens. The most useful feature of vertex form is that it shows the vertex of the parabola, which is the minimum value of y when $a > 0$ or the maximum value of y when $a < 0$. When the equation is not in vertex form, you'll have to do some work to find the vertex.

Here's a typical question about the vertex of a parabola.

 Mark for Review

16. The function g is defined by $g(x) = -x^2 + 4x + 12$. For what value of x does $g(x)$ reach its maximum?

Here's How to Crack It

The question asks for the value when a quadratic function reaches its maximum. A parabola reaches its minimum or maximum value at its vertex, so find the x -coordinate of the vertex. The simplest method is to enter the equation into the built-in calculator, then scroll and zoom as needed to find the vertex. The vertex is at $(2, 16)$, so the value of the x -coordinate is 2.

There is also a way to find the x -coordinate using algebra. In vertex form, the vertex is at (h, k) . When a quadratic is in standard form, $ax^2 + bx + c$, use the formula $h = -\frac{b}{2a}$ to find the x -coordinate of the vertex. The equation in this question is in standard form, so $a = -1$, $b = 4$, and $c = 12$. The formula becomes $h = -\frac{4}{2(-1)}$, or $h = -\frac{4}{-2}$, and then $h = 2$. You don't need this formula to answer this question, but if you needed to solve for the y -coordinate of the vertex, you could plug $x = 2$ back into the equation to get $-(2)^2 + 4(2) + 12 = -4 + 8 + 12 = 16$. This is the vertex of $(2, 16)$ that you found using the calculator.

Using either method, the correct answer is 2.



In the chapter on algebra, we showed you how the Digital SAT asks questions about the number of solutions to a system of linear equations or a quadratic. Review that to know the rules and how to use the discriminant. However, the calculator provides additional options, and some of the other skills you've learned in this book apply to questions about graphing too.

Think about using a calculator and apply what you've learned so far to the following question.

 Mark for Review

$$\frac{y}{3} = -\frac{7}{2}x$$

$$y = ax^2 + 12.25$$

17. In the system of equations above, a is a constant. If the system of equations has exactly one solution, what is the value of a ?

☐ A -2.25

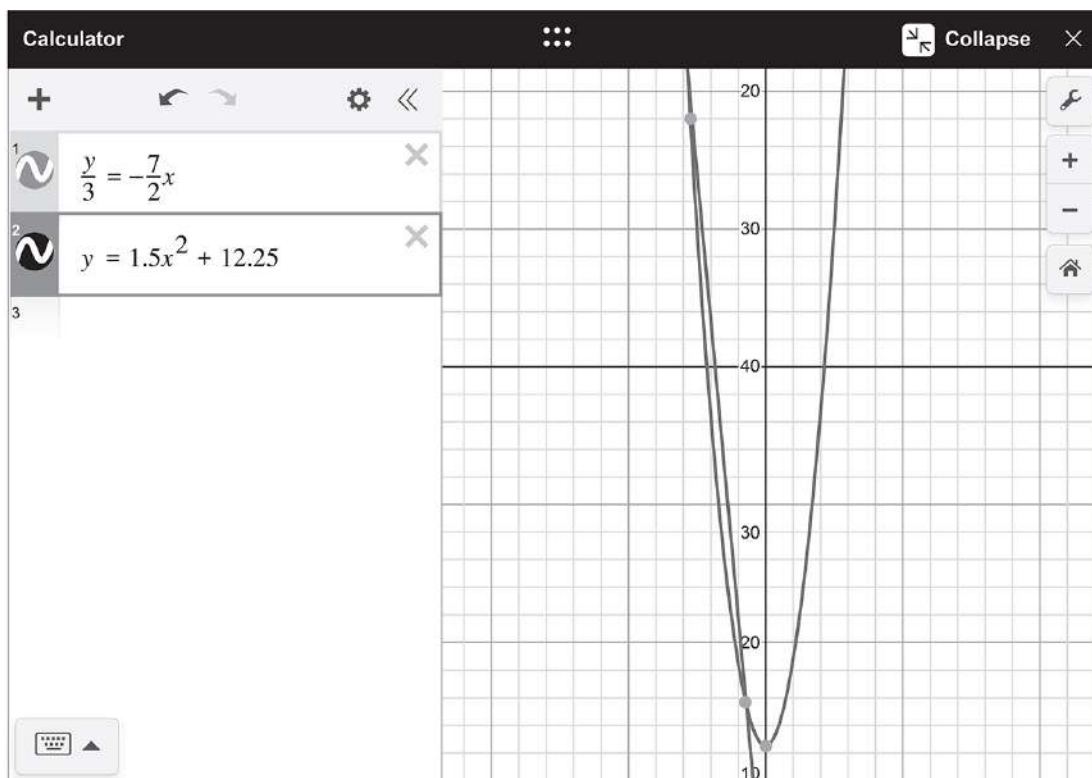
☐ B -1.5

☐ C 1.5

☐ D 2.25

Here's How to Crack It

The question asks for the value of a constant in a system of equations. The question asks for a specific value, and the answers contain numbers in increasing order, so plug in the answers. Yes, PITA can work on graphing questions, too! Start with an answer in the middle. Try (C), 1.5. Plug $a = 1.5$ into the second equation to get $y = (1.5)(x^2) + 12.25$. Next, enter both equations in the entry fields of the built-in graphing calculator and look at the results in the graph display. Zoom out enough to see both the parabola and the line clearly in the graph display (expanding the calculator window also helps). If you click one of the equations, several gray dots will appear on the graph. One shows the vertex of the parabola, and the other two show where the line intersects the parabola.



The question states that the *system of equations has exactly one solution*, not two, so eliminate (C). Next, try (D), 2.25. Keep the two equations you already entered and change 1.5 to 2.25 in the second equation. Click on one of the equations to see two gray dots: one is the vertex and the other is the single point where the line intersects the graph. The system now has exactly one solution, and the correct answer is (D).

EQUATION OF A CIRCLE

The Digital SAT will also ask questions about the equation of a circle in the xy -plane.

The equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

In the circle equation, the center of the circle is the point (h, k) , and the radius of the circle is r .

Let's look at a question that tests the use of the circle equation.

 Mark for Review

18. Circle P in the xy -plane has the equation $(x - 4)^2 + (y + 1)^2 = 9$. Circle Q has the same radius as Circle P , but its center is 2 units to the left of that of Circle P . Which of the following is the equation of Circle Q ?

☐ (A) $(x - 4)^2 + (y + 1)^2 = 1$

☐ (B) $(x - 2)^2 + (y + 1)^2 = 9$

☐ (C) $(x - 6)^2 + (y + 1)^2 = 9$

☐ (D) $(x - 4)^2 + (y + 3)^2 = 9$

Here's How to Crack It

The question asks for an equation that represents a graph. The equation of a circle in standard form is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. Since the equation of Circle P is $(x - 4)^2 + (y + 1)^2 = 9$, the center is at $(4, -1)$ and $r^2 = 9$, so $r = 3$. The question states that *Circle Q has the same radius as Circle P* , so the right side of the equation of Circle Q should also be 3^2 , or 9; eliminate (A) because it has an incorrect value for r^2 . The center of Circle Q is 2 units to the left of the center of Circle P . Two units to the left of $(4, -1)$ is $(2, -1)$, so the equation of Circle Q is $(x - 2)^2 + (y + 1)^2 = 9$. The correct answer is (B).

On the last question, you could have graphed the original circle and the circles in the answers. That would be time-consuming, though, and it was easy to find the center given the equation in standard form. Some circle questions are better cracked using a graphing calculator, either the built-in calculator or your own. Try using a calculator to answer this question.

 Mark for Review

19. What is the radius of a circle graphed in the xy -plane and given by the equation $3x^2 + 15x + 3y^2 - 3y = 28.5$?

☐ A 3

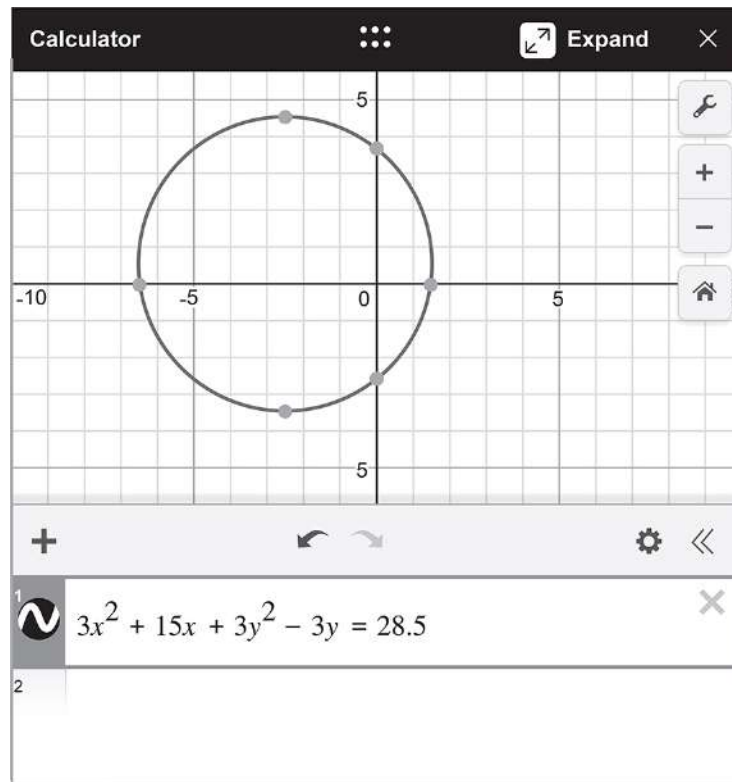
☐ B 4

☐ C $\sqrt{35}$

☐ D $\sqrt{48}$

Here's How to Crack It

The question asks for the radius of a circle given an equation for its graph. The equation is not given in standard form, so you cannot simply look at the equation and determine the radius. One approach is to graph the equation. Using the built-in calculator, enter the equation as given; the calculator will graph the circle correctly even though the equation isn't in standard form. Click in the entry field or on the circle to see several gray dots.



Click the dots at the maximum and minimum y -values to find the two ends of the diameter. The maximum y -value is at $(-2.5, 4.5)$, and the minimum y -value is at $(-2.5, -3.5)$. Because these points have the same x -coordinate, the distance between them is the diameter of the circle. Find the difference to get a diameter of $4.5 - (-3.5) = 8$. The radius of a circle is half the diameter, so the radius is 4.

The other way to solve for the radius is to convert the equation to standard form. The equation in standard form of a circle in the xy -plane is

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle and r is the radius. The equation given in the question is not in standard form. To put it into standard form, complete the squares. Start by dividing the entire equation by 3 to remove the coefficients on the x^2 and y^2 terms. This leaves

$$x^2 + 5x + y^2 - y = 9.5$$

Next, take half the coefficient on the x term, or 2.5, square it to get 6.25, and add 6.25 to both sides. Now the equation is

$$x^2 + 5x + 6.25 + y^2 - y = 15.75$$

Do the same with the coefficient on the y term to get

$$x^2 + 5x + 6.25 + y^2 - y + 0.25 = 16$$

The standard form equation for this circle is thus

$$(x + 2.5)^2 + (y - 0.5)^2 = 16$$

The question didn't ask for the center of the circle, so focus on the radius. If $r^2 = 16$, then $r = 4$.

Using a graphing calculator or completing the square, the correct answer is (B).



Got all of that? Now test your knowledge of functions and graphing with the following drills.

Functions and Graphs Drill

Use your new knowledge of functions, graphs, and coordinate geometry to complete the questions. Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. Function f is defined by $f(x) = x^2 - c$, where c is a constant. If $f(-2) = 6$, what is the value of c ?

☐ A -10

☐ B -2

☐ C 0

☐ D 2

 Mark for Review

2. A farming crew harvests 100 hectares a day on a wheat farm. If the crew maintains this rate, which of the following functions represents the number of hectares, h , the crew can harvest in d days?

☐ A $h(d) = d + 100$

☐ B $h(d) = d - 100$

☐ C $h(d) = 100d$

☐ D $h(d) = \frac{d}{100}$

 Mark for Review

x	y
-3	-7
-1	-3
2	3

3. Based on the chart above, which of the following could express the relationship between x and y ?

☐ A $y = x - 4$

☐ B $y = 2x - 1$

☐ C $y = 2x + 2$

☐ D $y = 3x - 3$

 Mark for Review

4. Line l contains points $(3, 2)$ and $(4, 5)$. If line m is perpendicular to line l , which of the following could be the equation of line m ?

☐ A $-5x + y = \frac{1}{3}$

☐ B $x + 3y = 15$

☐ C $x + 5y = 15$

☐ D $3x + y = 5$

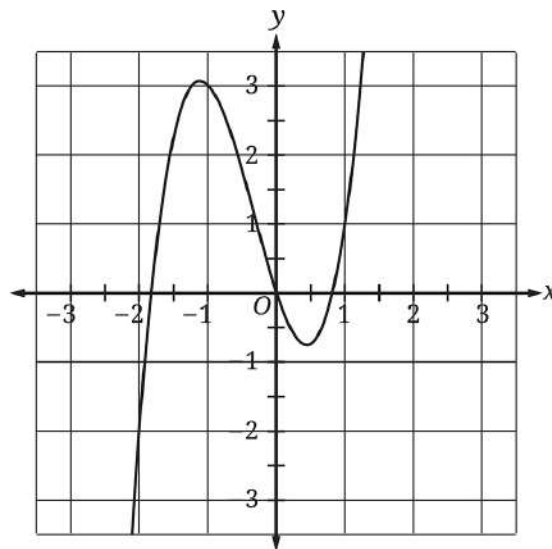
 Mark for Review

5. If $f(x) = 2x^2 + 4$ for all real numbers x , what is the value of $f(3) + 3f(5)$?

 Mark for Review

6. The variable c represents a positive constant in the exponential function $f(x) = \frac{1}{3}c^x$. What is the value of $f(3)$ if $f(4) = 27$?

 Mark for Review



7. The graph of $y = g(x)$ in the xy -plane is shown above. The function g is defined by $g(x) = ax^3 + bx^2 - cx$, where a , b , and c are constants. For how many values of x does $g(x) = 0$?

Zero

☐ B One

☐ C Two

☐ D Three

 Mark for Review

8. The acceleration of a ball rolling down a ramp can be estimated using the function $s(t) = 5t$, where s represents the speed of the ball and t represents time. Given this relationship, which of the following tables contains 4 values of t and the corresponding values of s ?

☐ A

t	s
1	5
2	10
3	15
4	20

☐ B

t	s
1	5
2	10
3	30
4	120

☐ C

t	s
1	5

2	25
3	125
4	625

D

t	s
5	1
10	2
15	3
20	4

 Mark for Review

9. The graph of line l in the xy -plane passes through the points $(2, 5)$ and $(4, 11)$. The graph of line m has a slope of -2 and an x -intercept of 2 . If point (x, y) is the point of intersection of lines l and m , what is the value of y ?

A $\frac{3}{5}$

B $\frac{4}{5}$

C 1

D 2

 Mark for Review

10. The movement of a roller coaster cart as it completes one ride can be modeled by the equation $y = -0.05(x - 55.5)^2 + 154$, which shows the

cart's height above the ground y , in feet, x seconds after the ride starts, where $0 < x \leq 111$. Which of the following statements best describes the meaning of the vertex of the graph of the equation in the xy -plane?

☐ A The cart reached a maximum height of 154 feet above the ground.

☐ B The cart reached a maximum height of 55.5 feet above the ground.

☐ C The cart's final height was 154 feet above the ground.

☐ D The cart's final height was 55.5 feet above the ground.

FUNCTIONS AND GRAPHS DRILL ANSWERS AND EXPLANATIONS

1. B

The question asks for the value of c . Start by plugging in what you know into the given function. If $f(x) = x^2 - c$, and $f(-2) = 6$, then plug in -2 for x in the function: $f(-2) = (-2)^2 - c$. Solve and replace $f(-2)$ with 6: $6 = 4 - c$; $2 = -c$; and $c = -2$. If you picked (A), you forgot that $(-2)^2$ is positive 4. The correct answer is (B).

2. C

The question asks for a function that represents a specific situation.

There are variables in the answer choices, and the question asks about the relationship between the number of hectares and the number of days, so plug in. Make $d = 2$. If the crew harvests 100 hectares in 1 day, it will harvest twice as many hectares, 200, in 2 days. Now plug d

$= 2$ and $h(d) = 200$ into the answer choices and eliminate any that don't work. Choice (A) becomes $200 = 2 + 100$, or $200 = 102$. This is not true, so eliminate (A). Choice (B) becomes $200 = 2 - 100$, or $200 = -98$; eliminate (B). Choice (C) becomes $200 = 100(2)$, or $200 = 200$; this is true, so keep (C) but check (D) just in case. Choice (D) becomes $200 = \frac{2}{100}$; eliminate (D). The correct answer is (C).

3. **B**

The question asks for the equation that best models pairs of values given in a table. Plug in the values from the chart. Use the pair $(-3, -7)$ from the top of the chart and eliminate answers that are not true. Choice (A) becomes $-7 = -3 - 4$, which is true. Keep it. Keep (B): $-7 = 2(-3) - 1$ is true. Get rid of (C), which becomes $-7 = 2(-3) + 2$: -7 does not equal -4 . Get rid of (D): $-7 = 3(-3) - 3$, and -7 does not equal -12 . Now use another pair just to test (A) and (B). Using $(-1, -3)$, (A) gives $-3 = -1 - 4$, which is not true, so eliminate it. The correct answer is (B).

4. **B**

The question asks for the equation of line m . First, find the slope of line l by using the slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - 3} = \frac{3}{1}$. A line perpendicular to line l must have a slope that is the negative reciprocal of l 's slope. So, its slope should be $-\frac{1}{3}$. In the standard form of a line $Ax + By = C$, the slope is $-\frac{A}{B}$. Only (B) has a slope of $-\frac{1}{3}$. If you

didn't remember the rule about the slope of perpendicular lines in standard form, you could have converted the answers to slope-intercept form and sketched out each of the lines to look for the answer that looked perpendicular to l . The correct answer is (B).

5. **184**

The question asks for the value of expression that contains two functions. In function notation, the number inside the parentheses is the x -value that goes into the function, or the input, and the value that comes out of the function is the y -value, or the output. When there is a number in front of the function, the output value is multiplied by that number. Plug $x = 3$ into the function to get $f(3) = 2(3)^2 + 4$, then $f(3) = 2(9) + 4$. This becomes $f(3) = 18 + 4$, and then $f(3) = 22$. Plug $x = 5$ into the function to get $f(5) = 2(5)^2 + 4$, then $f(5) = 2(25) + 4$, then $f(5) = 50 + 4$, and then $f(5) = 54$. Multiply this output by 3 to get $(3)(54) = 162$. Finally, add the values of $f(3)$ and $3f(5)$ to get $22 + 162 = 184$. The correct answer is 184.

6. **9**

The question asks for the value of a function. In function notation, $f(x) = y$. The number inside the parentheses is the x -value that goes into the function, or the input, and the value that comes out of the function is the y -value, or the output. According to the question, $f(4) = 27$, so when the input is 4, the output is 27. Plug in $f(x) = 27$ and $x = 4$, and solve for c . The function becomes $27 = \frac{1}{3}c^4$. Multiply both sides of the

equation by 3 to get $81 = c^4$. Take the fourth root of both sides of the equation to get $c = 3$. With the built-in calculator, enter $81^{\frac{1}{4}}$ or use the



button under the Functions menu to take the fourth root of 81.

The question asks for the value of $f(3)$, so plug $x = 3$ and $c = 3$ into the function, and solve for $f(3)$. The function becomes $f(3) = \frac{1}{3}(3^3)$, then $f(3) = \frac{1}{3}(27)$, which becomes $f(3) = 9$. The correct answer is 9.

7. **D**

The question asks for the number of times the value of a function is 0. In function notation, $f(x) = y$. The number inside the parentheses is the x -value that goes into the function, or the input, and the value that comes out of the function is the y -value, or the output. When $g(x) = 0$, the y -value on the graph is 0, so this question is asking for the number of x -intercepts. Look at the graph to see that it crosses the x -axis 3 times, with points at approximately $(-1.8, 0)$, $(0, 0)$, and $(0.8, 0)$. Thus, there are 3 times when $g(x) = 0$. The correct answer is (D).

8. **A**

The question asks for correct values in a function. In function notation, the number inside the parentheses is the x -value that goes into the function, or the input, and the value that comes out of the function is the y -value, or the output. In this scenario, t is the input and s is the output. When given a function and asked for the table of values, plug values from the answer choices into the function and eliminate answers that don't work. Start with $t = 2$. Plug $t = 2$ into the function to get $s(2) = 5(2)$, or $s(2) = 10$. Eliminate (C) because it has an incorrect value of s when t is 2. Plug $t = 3$ into the function to get $s(3) = 5(3)$, or $s(3) = 15$. Eliminate (B) because it has an incorrect

value of s when t is 3. Next, try a value for t that is in (D) but not (A). Plug $t = 5$ into the function to get $s(5) = 5(5)$, or $s(5) = 25$. Eliminate (D) because it has an incorrect value of s when t is 5. The correct answer is (A).

9. **D**

The question asks for the value of y . First, find the slope of line l by using the slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 5}{4 - 2} = \frac{6}{2} = 3$. Plug this slope and one of the points on line l into the slope-intercept form $y = mx + b$ to solve for b , giving you the full equation of the line. If you use the point $(2, 5)$, you get $5 = 3(2) + b$ or $5 = 6 + b$, so $b = -1$. Therefore, the equation for line l is $y = 3x - 1$. For line m , the slope is given as -2 , and the x -intercept is 2. Be very careful not to jump to the conclusion that the equation of line m is $y = -2x + 2$. In the form $y = mx + b$, the b is the y -intercept, not the x -intercept. The x -intercept is where $y = 0$, so you know that $(2, 0)$ is a point on line m . Use this point and the slope to find the equation of line m in the same way you did for line l : $0 = -2(2) + b$, so $b = 4$ and the equation is $y = -2x + 4$. Now set the x parts of the equations equal to find the point of intersection. If $3x - 1 = -2x + 4$, then $5x = 5$ and $x = 1$. Again, be careful! The question asked for the value of y . Plug $x = 1$ into one of the line equations to find y . For line l , the equation becomes $y = 3(1) - 1 = 3 - 1 = 2$. The correct answer is (D).

10. A

The question asks about the graph of the data representing a certain situation. Start by reading the final question, which asks for the meaning of the vertex of the graph. The function is a quadratic in vertex form, $y = a(x - h)^2 + k$, in which the vertex is at (h, k) . Since $a = -0.05$, the parabola opens downward, and the vertex of the parabola is at $(55.5, 154)$. Entering the equation into a graphing calculator is a good way to visualize all of this information. Next, label parts of the equation given. The question states that y is the height above the ground, in feet, and x is the number of seconds after the ride starts. The greatest height is at the vertex, and the y -value of the vertex is 154, so it is the maximum height. The correct answer is (A).

Summary

- Given a function, you put an x -value in and get an $f(x)$ or y -value out.
- Look for ways to use Plugging In and PITA on function questions.
- For questions about the graphs of functions, remember that $f(x) = y$.
- If the graph contains a labeled point or the question gives you a point, plug it into the equations in the answers and eliminate any that aren't true.
- The equation of a line can take two forms. In either form, (x, y) is a point on the line.
 - In slope-intercept form, $y = mx + b$, the slope is m and the y -intercept is b .
 - In standard form, $Ax + By = C$, the slope is $-\frac{A}{B}$ and the y -intercept is $\frac{C}{B}$.
- Given two points on a line, (x_1, y_1) and (x_2, y_2) , the slope is $\frac{(y_2 - y_1)}{(x_2 - x_1)}$.
- Two linear equations with infinitely many solutions represent the same line.
- Parallel lines have the same slopes and no points of intersection.
- Perpendicular lines have slopes that are negative reciprocals and intersect at a right angle.
- To find a point of intersection, plug the point into both equations to see if it works, or graph the lines on your calculator when it is allowed.

- The roots of a function, also known as solutions, zeroes, or x -intercepts, are the points where the graph crosses the x -axis and where $y = 0$.
- Graphs of functions can be moved up or down if a number is added to or subtracted from the function, respectively. They move left if a number is added inside the parentheses of the function or move right if a number is subtracted inside the parentheses.
- The standard form of a parabola is $y = ax^2 + bx + c$, where c is the y -intercept. If a is positive, the parabola opens up, and if a is negative, it opens down.
- The factored form of a parabola equation is $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the roots or x -intercepts of the parabola.
- The vertex form of a parabola equation is $y = a(x - h)^2 + k$, where (h, k) is the vertex. To get a parabola in the standard form into vertex form, complete the square.
- The standard form of a circle equation is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. To get a circle equation into the standard form, complete the square for both the x -terms and the y -terms.
- A graphing calculator can make an enormous difference on questions about graphs. Practice with the calculator you plan to use on test day—either the built-in calculator or your own—to learn all of the tricks and feel comfortable with using it during the test.

Chapter 24

Advanced Arithmetic

The Digital SAT includes questions about what the test-writers call Problem Solving and Data Analysis. Many of these questions test concepts you learned a few years ago, such as averages and proportions. More difficult questions will build on these basic concepts by requiring you to use charts and data or to combine multiple techniques. In this chapter, we will review the arithmetic and statistical concepts you'll need to know for the Digital SAT.

CALL ON THE CALCULATOR

While you work through the topics and examples in this chapter, don't forget to use the built-in calculator or your own. Make sure to review the Calculator Guide in your Student Tools so you can take advantage of this tool. Many of the examples in this chapter will show you how to do the question by hand because that can help make the concept clearer, but a calculator will help you do some questions very quickly while avoiding calculation errors. Even scary-sounding topics like margin of error come down to the basics of addition, subtraction, multiplication, and division, and that's what calculators do best!

RATIOS AND PROPORTIONS

A Ratio Is a Comparison

Many students get extremely nervous when they are asked to work with ratios. But there's no need to be nervous. A **ratio** is a comparison between the quantities of ingredients you have in a mixture, be it a class full of people or a bowl of cake batter. Ratios can be written to look like fractions—don't get them confused.

The ratio of x to y can be expressed in the following three ways:

1. $\frac{x}{y}$

2. the ratio of x to y

3. $x:y$

Part, Part, Whole

Ratios are a lot like fractions. In fact, anything you can do to a fraction (convert it to a decimal or percentage, reduce it, and so on), you can do to a ratio. The difference is that a fraction gives you a part (the top number) over a whole (the bottom number), while a ratio typically gives you two parts (boys to girls, cars to trucks, sugar to flour), and it is your job to come up with the whole. For example, if there is one cup of sugar for every two cups of flour in a recipe, that's three cups of stuff. The ratio of sugar to flour is 1:2. Add the parts to get the whole.

Ratios vs. Fractions

Keep in mind that a ratio compares part of something to another part. A fraction compares part of something to the whole thing.

Ratio: $\frac{\text{part}}{\text{part}}$

Fraction: $\frac{\text{part}}{\text{whole}}$

Ratio to Real

If a class contains 3 students and the ratio of boys to girls in that class is 2:1, how many boys and how many girls are there in the class? Of course, there are 2 boys and 1 girl.

Now, suppose a class contains 24 students and the ratio of boys to girls is still 2:1. How many boys and how many girls are there in the class? This is a little harder, but the answer is easy to find if you think about it. There are 16 boys and 8 girls.

How did we get the answer? We added up the number of “parts” in the ratio (2 parts boys plus 1 part girls, or 3 parts all together) and divided it into the total number of students. In other words, we divided 24 by 3. This told us that the class contained 3



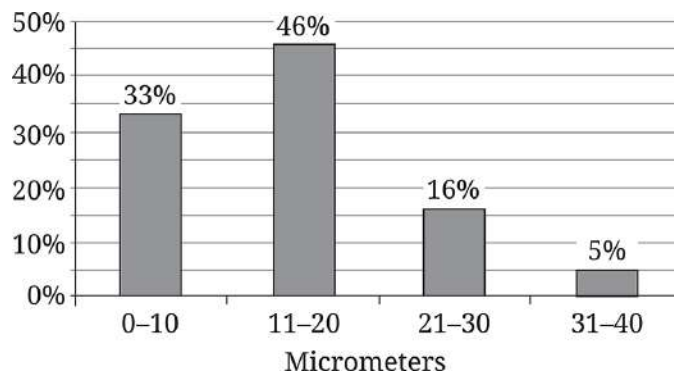
equal parts of 8 students each. From the given ratio (2:1), we knew that two of these parts consisted of boys and one of them consisted of girls.

The test-writers will often combine ratios with diagrams or data from charts and graphs. Don't let them intimidate you with these: just work in bite-sized pieces and write down the part-to-part relationships that you need to solve the question.

Try this example.

 Mark for Review

1. A lapping slurry contains microbeads suspended in a solution and is used to polish a silicon wafer by abrasion of the surface. The distribution of the particle size, in micrometers, is shown below.



Which of the following is closest to the ratio of the number of 11–20 micrometer microbeads to the number of 31–40 micrometer microbeads?

☐ A 1:9

☐ B 2:1

☐ C 3:1

☐ D 9:1

Here's How to Crack It

The question asks for a ratio based on data from a graph. To find the ratio of 11–20 micrometer microbeads to 31–40 micrometer microbeads, read carefully, look up the right information in the graph, and set up the part-to-part relationship. The 11–20 micrometer microbeads make up 46% of the total, and the 31–40 micrometer microbeads make up 5% of the total. Plug in 100 for the total to get actual numbers of 46 and 5, respectively. That is a ratio of 46:5. Because the question is asking for the closest ratio, round the numbers to get a ratio of 45:5, which reduces to 9:1. Notice that (A) has the right numbers in the wrong order; always read carefully and use your scratch paper to avoid this mistake. The correct answer is (D).

Proportions Are Equal Ratios

Some Digital SAT math questions will contain two proportional, or equal, ratios from which one piece of information is missing.

Here's an example.

 Mark for Review

2. If 2 packages contain a total of 12 doughnuts, how many doughnuts are there in 5 packages?

☐ A 24

☐ B 30

☐ C 36

Here's How to Crack It

The question asks for the number of doughnuts in 5 packages. This question simply describes two equal ratios, one of which is missing a single piece of information. Here's the given information represented as two equal ratios:

$$\frac{2 \text{ (packages)}}{12 \text{ (doughnuts)}} = \frac{5 \text{ (packages)}}{x \text{ (doughnuts)}}$$

Because ratios can be written so they look like fractions, you can treat them exactly like fractions. To find the answer, all you have to do is solve for x . Start by cross-multiplying:

$$\frac{2}{12} \swarrow \searrow \frac{5}{x}$$

$$\text{so, } 2x = 60$$

$$x = 30$$

The correct answer is (B).

Proportions: Advanced Principles

Many proportion questions will also involve unit conversion. Be sure to pay attention to the units and have the same units in both numerators and the same units in both denominators.

Let's look at an example.

3. Gary is using a 3D printer to create a miniature version of himself. The scale of the miniature is 0.4 inches to 1 foot of Gary's actual height. If Gary is 5 feet and 9 inches tall, what will be the height of his 3D-printed miniature? (12 inches = 1 foot)

☐ A 2.0 inches

☐ B 2.3 inches

☐ C 2.6 inches

☐ D 2.9 inches

Here's How to Crack It

The question asks for the height of the 3D miniature. The scale of the 3D printer is in inches and feet—0.4 inches on the miniature for every 1 foot in real life. Start by converting every measurement to inches. There are 12 inches in each foot, so the scale will be 0.4 inches = 12 inches in real life. Now convert Gary's height into inches. Begin by setting up a proportion to find out how many inches are in 5 feet.

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x \text{ inches}}{5 \text{ feet}}$$

Cross-multiply to find that 5 feet equals 60 inches. Gary is 5 feet and 9 inches tall, so he is $60 + 9 = 69$ inches tall. Now set up a proportion with the scale of the miniature and Gary's height in inches.

$$\frac{0.4 \text{ inches}}{12 \text{ inches}} = \frac{x \text{ inches}}{69 \text{ inches}}$$

Cross-multiply to get $12x = 27.6$, and then divide both sides by 12 to find that $x = 2.3$ inches. The correct answer is (B).



Now try a question that requires converting the units of both parts of a rate.

 Mark for Review

4. A car is traveling on the highway at a speed of 65 miles per hour. Which of the following is the best approximation of the car's speed, in feet per second? (1 mile = 5,280 feet)

☐ A 95

☐ B 190

☐ C 5,720

☐ D 343,200

Here's How to Crack It

The question asks for a rate in different units. Take it one piece at a time to avoid getting confused. Start by converting miles to feet. The question states that *1 mile = 5,280 feet*, so set up a proportion to determine how many feet are in 65 miles, being sure to match up units. The proportion is $\frac{1 \text{ mile}}{5,280 \text{ feet}} = \frac{65 \text{ miles}}{x \text{ feet}}$. Cross-multiply to get $(5,280)(65) = x$, or $x = 343,200$ feet per hour. This is answer choice (D), but you're only halfway done with the conversion. Eliminate trap answer (D).

Next, convert hours to seconds. There are 60 minutes in 1 hour and 60 seconds in 1 minute, so there are $(60)(60) = 3,600$ seconds in 1 hour. Thus,

a rate of $\frac{343,200 \text{ feet}}{1 \text{ hour}}$ is the same as a rate of $\frac{343,200 \text{ feet}}{3,600 \text{ seconds}}$. Reduce the fraction to get a rate of $95.\overline{3}$ feet per second. The question asks for the *best approximation of the car's speed*, and the closest answer is 95. The correct answer is (A).

PERCENTAGES

Percentages Are Fractions

There should be nothing frightening about a percentage. It's just a convenient way of expressing a fraction with a denominator of 100.

Percent means “per 100” or “out of 100.” If there are 100 questions on your math test and you answer 50 of them, you will have answered 50 out of 100, or $\frac{50}{100}$, or 50 percent. To think of it another way:

$$\frac{\text{part}}{\text{whole}} = \frac{x}{100} = x \text{ percent}$$

Memorize These Percentage-Decimal-Fraction Equivalents

These show up all the time, so go ahead and memorize them.

$$0.01 = \frac{1}{100} = 1 \text{ percent}$$

$$0.1 = \frac{1}{10} = 10 \text{ percent}$$

$$0.2 = \frac{1}{5} = 20 \text{ percent}$$

$$0.25 = \frac{1}{4} = 25 \text{ percent}$$

$$0.5 = \frac{1}{2} = 50 \text{ percent}$$

$$0.75 = \frac{3}{4} = 75 \text{ percent}$$

Converting Percentages to Fractions

To convert a percentage to a fraction, simply put the percentage over 100 and reduce:

$$80 \text{ percent} = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

Converting Fractions to Percentages

Because a percentage is just another way to express a fraction, you shouldn't be surprised to see how easy it is to convert a fraction to a percentage. To do so, simply use a calculator to divide the top of the fraction by the bottom of the fraction, and then multiply the result by 100. Here's an example:

Problem: Express $\frac{3}{4}$ as a percentage.

Solution: $\frac{3}{4} = 0.75 \times 100 = 75 \text{ percent}.$

Converting fractions to percentages is easy with the built-in calculator or your own. In fact, a calculator is going to be the fastest and most accurate way to answer most questions about percentages.

Converting Percentages to Decimals

To convert a percentage to a decimal, simply move the decimal point *two places to the left*. For example, 25 percent can be expressed as the decimal 0.25; 50 percent is the same as 0.50 or 0.5; 100 percent is the same as 1.00 or 1.



Call on the Calculator

The built-in calculator can give you an average at the click of a button. Read the section on the scientific calculator in the Digital SAT Calculator Guide in your Student Tools to review how to find that button in the Functions menu and how to use it to work with averages.

Converting Decimals to Percentages

To convert a decimal to a percentage, just do the opposite of what you did in the preceding section. All you have to do is move the decimal point *two places to the right*. Thus, $0.5 = 50$ percent; $0.375 = 37.5$ percent; $2 = 200$ percent.

The following drill will give you practice working with fractions, decimals, and percentages.

FRACTIONS, DECIMALS, AND PERCENTS EXERCISE

Fill in the missing information in the following table. Answers can be found on [this page](#).

	Fraction	Decimal	Percent
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	$\frac{1}{5}$	0.2	20%
<u>1.</u>	$\frac{1}{2}$		
<u>2.</u>		3.0	
<u>3.</u>			0.5%
<u>4.</u>	$\frac{1}{3}$		

Translation, Please!

Word problems can be translated into arithmetic symbols. Learning how to translate from English to math will help you immensely on the Digital SAT Math section. We covered some of this already, but as a review, here are some of the most common terms you will see in word problems and their math symbol equivalents:

Word	Symbol
is, are, costs	=
greater than, more than	+
fewer than, less than	−
of	× (multiply)
percent	÷ 100
what	n (variable)

Do You Speak Math?

Problem: What number is 5 more than 10 percent of 20?

Students often make careless errors on questions like this because they aren't sure how to translate the words they are reading into math. You won't make mistakes if you take the words slowly, one at a time, and translate each one into a mathematical symbol. Use the chart above to write this question in math. *What number* means “variable,” so you can write that as n (or x or whatever letter works for you). *Is* means “equals,” so now you have $n =$. Next you are given the number 5, so write that in your equation and you get $n = 5$. *More than* translates to $+$, and *10 percent* is $\frac{10}{100}$. That gives you $n = 5 + \frac{10}{100}$. Finally, *of 20* means multiply by 20, so now you have the equation:

$$n = 5 + \frac{10}{100}(20)$$

$$n = 5 + 2$$

$$n = 7$$

You will see the words *of*, *is*, *product*, *sum*, and *what* pop up a lot in math questions on the Digital SAT. Don't let these words fool you because they all translate into simple math functions. Memorize all of these terms and their math equivalents. It will save you time on the test and make your life with the Digital SAT much less unpleasant.

What Percent of What Percent of What?

On more challenging Digital SAT questions, you may be asked to determine the effect of a series of percentage increases or decreases. The key point to

remember on such questions is that each successive increase or decrease is performed on the result of the previous one.

Here's an example.

 Mark for Review

5. A business paid \$300 to rent a piece of office equipment for one year. The rent was then increased by 10% each year thereafter. How much will the company pay for the first three years it rents the equipment?

☐ A \$920

☐ B \$960

☐ C \$990

☐ D \$993



Bite-Sized Pieces

Always handle percentage problems using Bite-Sized Pieces: one piece at a time.

Here's How to Crack It

The question asks for the cost of the equipment over three years. This question is a great place to use the Bite-Sized Pieces strategy. You know

that the business paid \$300 to rent the piece of office equipment for the first year. Then, you were told that the rent increases by 10 percent for each year thereafter. That's a sure sign that you're going to need the rent for the second year, so go ahead and calculate it. For the second year, the rent is $300 + \left(\frac{10}{100} \times 300 \right) = 330$.

Now, the question tells you that the business rents the equipment for three years. So, you need to do the calculation one more time. At this point, you might want to set up a chart to help keep track of the information.

Year 1: \$300

Year 2: $300 + \left(\frac{10}{100} \times 300 \right) = \330

Year 3: $330 + \left(\frac{10}{100} \times 330 \right) = \363

To find the answer, all you need to do is add up the costs for each of the three years.

Year 1: \$300

Year 2: \$330

Year 3: \$363
\$993

The correct answer is (D).



What Percent of What Percent of...Yikes!

Sometimes you may find successive percentage questions in which you aren't given actual numbers to work with. In such cases, you need to plug in some numbers.

Here's an example.

 Mark for Review

6. A number is increased by 25 percent and then decreased by 20 percent. The result is what percent of the original number?

☐ A 80

☐ B 100

☐ C 105

☐ D 120

Here's How to Crack It

The question asks for the result of a percent increase and a percent decrease on an original number. You aren't given a particular number to work with in this question—just “a number.” Rather than trying to deal with the problem in the abstract, you should immediately plug in a number to work with. What number would be easiest to work with in a percentage question? Why, 100, of course.

1. 25 percent of 100 is 25, so 100 increased by 25 percent is 125.
2. Now you have to decrease 125 by 20 percent; 20 percent of 125 is 25, so 125 decreased by 20 percent is 100.

3. 100 (the result) is 100 percent of 100 (the number you plugged in). The correct answer is (B).



Plugging Away at Relationships

Questions dealing with percents, fractions, and other ways of expressing relationships among numbers are great chances to plug in!

Remember, never try to solve a percentage problem by writing an equation if you can plug in numbers instead. Using Plugging In on percentage questions is faster, easier, and more accurate. Why work through long, arduous equations if you don't have to?



AVERAGES

What Is an Average?

The **average**, also called the **arithmetic mean**, of a set of n numbers is simply the sum of all the numbers divided by n . In other words, if you want to find the average of three numbers, add them up and divide by 3. For example, the average of 3, 7, and 8 is $\frac{(3+7+8)}{3}$, which equals $\frac{18}{3}$, or 6.

That was an easy example, but average questions on the Digital SAT won't always have clear solutions. That is, you won't



always be given the information for averages in a way that is easy to work with. For that reason, use the formula $T = AN$, in which T is the *total*, A is the *average*, and N is the *number of things*. The total is the sum of all the numbers you're averaging, and the number of things is the number of elements you're averaging. Plug in the information you've been given, and then you can solve the equation for the quantity that you don't know.

Here's what the formula looks like using the simple average example we just gave you.

$$T = AN$$

$$3 + 7 + 8 = (A)(3)$$

$$18 = (A)(3)$$

$$A = 6$$

Here's another simple example:

Problem: If the mean of three test scores is 70, what is the total of all three test scores?

Solution: Just put the average (70) and the number of things (3 tests) into the formula to get $T = (70)(3)$. Then multiply to find the total, which is 210.

Total

When calculating averages and means, always find the total. It's the one piece of information that the Digital SAT loves to withhold.

Mark It!

Make sure you're using the formula $T = AN$ each time you see the word *mean* or *average* in a question.

Averages: Advanced Principles

To solve most difficult average questions, all you have to do is use the formula more than once. Most of the time you will use it to find the total of the number being averaged. Here's an example.

 Mark for Review

7. Maria has taken four chemistry tests and has a mean score of 80. If she scores a 90 on her fifth chemistry test, what is her mean for these five tests?

☐ **A** 80☐ **B** 81☐ **C** 82☐ **D** 84

Here's How to Crack It

The question asks for the mean score Maria received on all 5 tests. Start by writing out $T = AN$ and filling in what you know. You can put 80 in for the average and 4 in for the number of things to get $T = (80)(4)$. You can calculate that Maria has gotten 320 total points on her first four tests.

Now, since the question mentions another average, write the formula again and fill in the new information. This time, there are five tests, making the

formula $T = (A)(5)$. The question asks for the average, so you also need to find the total. The total for all five tests is the total from the first four tests plus the score from the fifth test: $320 + 90 = 410$. Put that into the formula to get $410 = (A)(5)$ and divide to find the average: 82. The correct answer is (C).

Averages, and many other arithmetic topics, may be tested using charts and data. To find the numbers to average, look them up on the graphic provided and watch out for mismatched units.

 Mark for Review

Charge No.	Battery Life
1	1:11
2	1:05
3	0:59
4	0:55
5	0:55
6	0:54
7	0:54

8. A toy drone is opened and charged to full battery life. The table above shows the duration of the battery life in hours and minutes between charges. What is the mean battery life for the first five charges?

55 minutes

☐ B 58 minutes

☐ C 1 hour and 1 minute

☐ D 1 hour and 5 minutes

Here's How to Crack It

The question asks for the mean battery life for the first 5 charges. To find the average, add up the battery life values for the first 5 charges and divide by 5. Make sure that you convert the battery charge time for charges 1 and 2 into minutes before calculating: $1:11 = 60 + 11 = 71$ minutes, and $1:05 = 60 + 5 = 65$ minutes. The average is equal to $\frac{71+65+59+55+55}{5} = \frac{305}{5} = 61$ minutes, which is equal to 1 hour and 1 minute. The correct answer is (C).



Call on the Calculator

Review the first two sections of the Digital SAT Calculator Guide in your Student Tools to remember how to work with fractions, decimals, and percentages using the calculator. If you're planning to use your own calculator, make sure you know how to use it to make these conversions.

Don't forget that you can also plug in when answering average questions.

 Mark for Review

9. The mean of a list of 5 numbers is n . When an additional number is added to the list, the mean of all 6 numbers is $n + 3$. Which of the following is the value, in terms of n , of the number added to the list?

☐ A $6n + 18$

☐ B $5n$

☐ C $n + 18$

☐ D $n + 6$

Here's How to Crack It

The question asks for the value of a number added to the list. There are variables in the answers, so plug in for the value of n , which is the average. If $n = 20$, then you can use the average formula to find the total of the five numbers on the list.

$$T = AN$$

$$T = (20)(5)$$

$$T = 100$$

A number is added and there is a new average, so it's time to write out the formula again. For this one, you know that there are 6 numbers and that their average is $n + 3 = 20 + 3 = 23$.

$$T = AN$$

$$T = (23)(6)$$

$$T = 138$$

Since the difference in the two totals was caused by the addition of the sixth number, the sixth number must be $138 - 100 = 38$. That's the target, so be sure to circle it. Now check the answer choices. Choice (A) becomes $6(20) + 18 = 120 + 18 = 138$. This does not match the target, so eliminate it. The value with (A) was much too large, so (B) will also be large. Try (C) and (D) next: (C) becomes $20 + 18 = 38$, and (D) becomes $20 + 6 = 26$. Only (C) matches the target, so the correct answer is (C).



On the Digital SAT, you'll also need to know four other statistical topics: *median*, *mode*, *range*, and *standard deviation*. These topics have pretty straightforward definitions. One way the Digital SAT will complicate the issue is by presenting the data in a chart or graph, making it harder to see the numbers you are working with.

WHAT IS A MEDIAN?

The **median** of a list of numbers is the number that is exactly in the middle of the list when the list is arranged from smallest to largest, as on a number line. For example, in the group 3, 6, 6, 6, 6, 7, 8, 9, 10, 10, 11, the median is 7. Five numbers come before 7 in the list, and 5 come after. Remember it this way: median sounds like *middle*.



Missing the Middle?

To find the median of a set containing an even number of items, take the average of the two middle numbers.

Let's see how this idea might be tested.

 Mark for Review

Milligrams of Gold					
	1	2	3	4	5
Limestone	0.45	0.58	0.55	0.42	0.41
Granite	0.94	0.87	0.82	0.55	0.73
Gneiss	0.38	0.60	0.37	0.40	0.34

10. Five samples of each of three different rock types were collected on a hiking trip in Colorado. Each sample was analyzed for its gold content. The milligrams of gold found in each sample are presented in the table above. How much larger is the median of the amount of gold in the granite samples than that of the limestone samples?

☐ A 0.00

☐ B 0.37

☐ C 0.45

☐ D 0.55

Here's How to Crack It

The question asks for a comparison of the medians of data for limestone and granite. Start by putting the gold weights for limestone in order to get

0.41, 0.42, 0.45, 0.55, 0.58

The median for limestone is the middle number: 0.45 mg.

Next, place the gold weights for granite in order to get

0.55, 0.73, 0.82, 0.87, 0.94

The median for granite is 0.82.

Therefore, the difference between the median amount of gold in the granite and limestone samples is $0.82 - 0.45 = 0.37$. The correct answer is (B).



WHAT IS A MODE?

The **mode** of a group of numbers is the number in the list that appears most often. In the list 3, 4, 4, 5, 7, 7, 8, 8, 8, 9, 10, the mode is 8, because it appears three times while no other number in the group appears more than twice. Remember it this way: *mode* sounds like “most.”

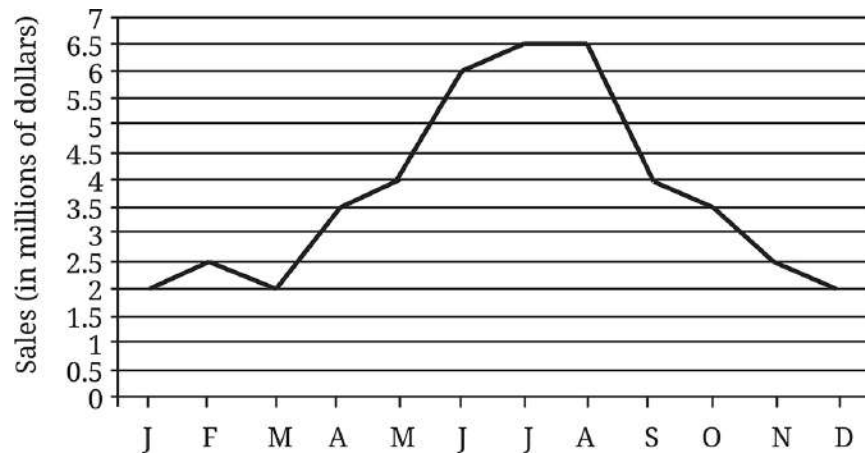
Because mode by itself is pretty easy to figure out, questions will usually combine mode with other concepts like mean, median, or range.

WHAT IS A RANGE?

The **range** of a list of numbers is the difference between the greatest number in the list and the least number in the list. For the list 4, 5, 5, 6, 7, 8, 9, 10, 20, the greatest number is 20 and the least is 4, so the range is $20 - 4 = 16$.



 Mark for Review



11. The forecasted monthly sales of a type of sunscreen are presented in the figure above. Which of the following best describes the range of monthly sales, in millions of dollars, throughout the year shown?

☐ A 2.5

☐ B 3.5

☐ C 4.0

☐ D 4.5

Here's How to Crack It

The question asks for the range in monthly sales based on the graph. The range of a set of values is the difference between the greatest and the smallest value. The lowest monthly sales number for the sunscreen can be found where the line dips closest to the bottom of the graph. This happens in both January and March, when the forecasted sales are 2 million. Make sure to read the units carefully. The highest point is where the line goes closest to the top of the graph. This happens in July and August, when the forecasted monthly sales are 6.5 million. Therefore, the range is 6.5 million – 2 million = 4.5 million. The correct answer is (D).

The Digital SAT might even have a question that tests more than one of these statistical concepts at the same time. Take it one step at a time and use POE when you can.

 Mark for Review

Precious Metals in Catalytic Converters, in grams					
1	2	2	3	4	6
6	6	9	9	10	10
11	13	14	14	15	17

12. The grams of precious metals in recycled catalytic converters were measured for a variety of automobiles. The data are presented in the table above. If the lowest data point, 1 gram, and highest data point, 17 grams, are removed from the set, which of the following quantities would change the most?

☐ A Mode

☐ B Mean

☐ C Range

☐ D Median

Here's How to Crack It

The question asks for the measure of the data that will change the most if two specific data points are removed. Start by evaluating the easier answer choices and save mean for last. The mode of the current list is 6, and removing 1 and 17 from the list won't change that. Eliminate (A). The range is the difference between the smallest number and the largest number

on the list. Right now, the range is $17 - 1 = 16$. If those extremes are removed from the list, the new range is $15 - 2 = 13$, and the range changed by 3 units. Keep (C) for now. The median is the middle number in the list, or the average of the middle two numbers. Currently, both middle numbers are 9, so the median is 9. This won't change if 1 and 17 are removed, so eliminate (D). The mean of a list is not likely to change dramatically with the removal of the numbers at the extremes, so (C) is likely correct. To actually evaluate the mean, you need to add up all the numbers on the list and divide by the number of items in the list. For the current list, the total is 152 for the 18 items, so the average is $8.\overline{44}$. To find the new total if 1 and 17 are removed, don't re-add everything; just subtract 18 from the previous total. The new list will have only 16 items, so the new average is 8.375. This is only slightly different than the previous mean, so eliminate (B). The correct answer is (C).

WHAT IS A MARGIN OF ERROR?

Some questions will test **margin of error**, which is a range of percentages rather than numbers. A margin of error gives a range for random sampling errors in a survey or poll. It indicates how much the results might change if the poll were repeated or if the entire population were asked instead of a random sample. For example, if a survey shows that 70% of randomly sampled test-takers prefer the Digital SAT to the paper-and-pencil SAT, and there is a margin of error of $\pm 5\%$, that means it is highly likely that between 65% and 75% of all test-takers prefer the Digital SAT.

Let's look at an example of how this can be tested.

 Mark for Review

13. A summer beach volleyball league has 750 players in it. At the start of the season, 150 of the players are randomly chosen and polled on whether games will be played while it is raining or if the games should be canceled. The results of the poll show that 42 of the polled players would prefer to play in the rain. The margin of error on the poll is $\pm 4\%$. What is the range of players in the entire league that would be expected to prefer to play volleyball in the rain rather than cancel the game?

☐ A 24–32

☐ B 39–48

☐ C 150–195

☐ D 180–240

Here's How to Crack It

The question asks for the range of players that would prefer to play in the rain. The first step is to determine the percent of polled players that wanted to play in the rain.

$$\frac{42}{150} = 0.28 \text{ or } 28\%$$

Now apply this percent to the entire population of the league. Since 28% of the polled players wanted to play in the rain, 28% of all players should want to play in the rain.

$$\frac{28}{100} \times 750 = 210$$

The only range that contains this value is (D), so that is the correct answer. To actually calculate the margin of error, add and subtract 4% to the actual percent of 28% to get a range of 24–32% of the total.

$$24\% \text{ of } 750 = 180$$

$$32\% \text{ of } 750 = 240$$

Therefore, the entire range is 180 to 240. The correct answer is (D).

WHAT IS A FREQUENCY TABLE?

Another way the Digital SAT tests statistical concepts is by using a **frequency table**. This is just what it sounds like: a table to show how frequently something happens. One column shows the numbers of something, like ages or scores on a test, and the other column shows how often that thing occurs. If you wanted to show, for example, how many meetings each member of a 15-person club attended, you could use the table below.

Meeting Attendance for this Cool Club I'm In

Meetings attended	Frequency
1	2
2	1
3	1
4	3
5	5
6	3

Frequency tables give you ways to find the mean, median, and mode of a list of numbers without needing to write out the whole list, so the test-writers will often combine those concepts with frequency tables in the same question.

Try out an example of this below.

 Mark for Review

14. Number of Ice Cream Scoops for Customers at an Ice Cream Parlor

Number of Ice Cream Scoops	Frequency
8	1
5	2
4	2
3	7
2	6
1	6

The distribution of ice cream scoops for 24 customers at an ice cream parlor is displayed in the table above. Which of the following orders the median, mode, and range correctly?

☐ A mode < median < range

☐ B median < mode < range

☐ C median < range < mode

☐ D range < median < mode

Here's How to Crack It

The question asks for the correct order of the median, mode, and range of the data from least to greatest. Start with the mode, which is easy to determine from the table. Since the greatest number in the Frequency

column is 7, the corresponding number of scoops is the mode. Thus, the mode is 3.

The range is also easy to determine from the table because the range of a list of numbers is the difference between the greatest number and the least number. The greatest number of ice cream scoops is 8 and the least number of ice cream scoops is 1, so the range is $8 - 1 = 7$. The mode of 3 is less than the range of 7. Eliminate (C) and (D) because they have the range as less than the mode.

Next, find the median, which is the middle number of an ordered list. You could write out all 24 numbers in this list and count up to the middle number(s), but there's a better way. When a list has an even number of terms, the median is the average of the two middle terms. Since $\frac{24}{2} = 12$, the median is the average of the 12th and 13th terms. To confirm this, notice that there are $12 - 1 = 11$ terms to the left of the 12th term and $24 - 13 = 11$ terms to the right of the 13th term. Now, use the frequency table to find the 12th and 13th terms. The table shows that 6 customers chose 1 scoop, and 6 customers chose 2 scoops, so the 12th term is 2. The next 7 customers chose 3 scoops, so the 13th term is 3. The average of 2 and 3 is $\frac{2+3}{2} = 2.5$. Thus, the median is 2.5. This is less than 3, so the median is less than the mode. Eliminate (A) because it has the mode as less than the median. The correct answer is (B).



WHAT IS STANDARD DEVIATION?

In real-world applications, **standard deviation** is a measure of how numbers are distributed around the mean, and the calculations can get complicated. But Digital SAT math is not the real world! Think of standard deviation as similar to range in that it shows the spread of a group of numbers. When the numbers are more spread out around the mean, the standard deviation is greater. When the numbers are clumped closer together around the mean, the standard deviation is smaller.

Take a look at the example on the next page of how a Digital SAT question might combine standard deviation with another statistical concept you already know.

 Mark for Review

Data Set 1	Data Set 2
6	4
4	3
3	7
8	8
4	2
6	5
5	6

15. The table above shows two sets of data. Of the following statements comparing Data Set 1 to Data Set 2, which is true?

- ☐ A The standard deviations are the same, and the medians are the same.

☐ B The standard deviations are the same, and the medians are different.

☐ C The standard deviations are different, and the medians are the same.

☐ D The standard deviations are different, and the medians are different.

Here's How to Crack It

The question asks which statement is true about the medians and standard deviations of two data sets. Start by finding the median of each data set, recalling that the median of a group of numbers is the middle number when all values are arranged in order. Start by putting the lists in order. Data Set 1 is 3, 4, 4, 5, 6, 6, 8, and Data Set 2 is 2, 3, 4, 5, 6, 7, 8. Both Data Set 1 and Data Set 2 have 7 numbers, so the median will be the fourth number. The median of Data Set 1 is 5, and the median of Data Set 2 is 5. The medians are the same, so eliminate (B) and (D).

Standard deviation is a measure of the spread of a group of numbers. In Data Set 1, the numbers are clustered toward the middle. In Data Set 2, each number appears once and the numbers are evenly spread throughout the list. Thus, Data Set 1 has a smaller standard deviation than does Data Set 2. Eliminate (A), which says the standard deviations are the same. The correct answer is (C).

PROBABILITY

Probability is a mathematical expression of the likelihood of an event. The basis of probability is simple. The likelihood of any event is discussed in terms of all of the possible outcomes. To express the probability of a given event, x , you would count the number of possible outcomes, count the

number of outcomes that give you what you want, and arrange them in a fraction, like this:

$$\text{Probability of } x = \frac{\text{number of outcomes that give you what you want}}{\text{total number of possible outcomes}}$$

Every probability is a fraction. The largest a probability can be is 1; a probability of 1 indicates total certainty. The smallest a probability can be is 0, meaning that it's something that cannot happen. Furthermore, you can find the probability that something WILL NOT happen by subtracting the probability that it WILL happen from 1. For example, if the meteorologist tells you that there is a 0.3 probability of rain today, then there must be a 0.7 probability that it won't rain, because $1 - 0.3 = 0.7$. Figuring out the probability of any single event is usually simple. When you flip a coin, for example, there are only two possible outcomes, heads and tails; the probability of getting heads is therefore 1 out of 2, or $\frac{1}{2}$. When you roll a die, there are six possible outcomes, 1 through 6; the odds of getting a 4 are therefore $\frac{1}{6}$. The odds of getting an even result when rolling a die are $\frac{1}{2}$ because there are 3 even results in 6 possible outcomes.

Here's an example of a probability question.



 Mark for Review

16. A bag contains 7 blue marbles and 14 marbles that are not blue. If one marble is drawn at random from the bag, what is the probability that the marble is blue?

☐ A $\frac{1}{3}$

☐ B $\frac{1}{2}$

☐ C $\frac{2}{3}$

☐ D $\frac{3}{7}$

Here's How to Crack It

The question asks for the probability that a selected marble is blue. To make the probability, find the number of blue marbles and the total number of marbles. Here, there are 21 marbles in the bag, 7 of which are blue. The probability that a marble chosen at random would be blue is therefore $\frac{7}{21}$, or $\frac{1}{3}$. The correct answer is (A).



Let's look at a probability question based on a chart. Again, getting to the correct answer involves reading the chart carefully to find the right numbers to use.



 Mark for Review

The table below shows the distribution of 300 high school students signing up for elective classes.

	Culinary Arts	World Literature	Computer Coding	Total
Freshmen	18	22	38	78
Sophomores	24	21	15	60
Juniors	30	9	18	57
Seniors	37	38	30	105
Total	109	90	101	300

17. If one of the students is randomly selected, what is the probability of selecting a student who is a junior taking culinary arts? (Express your answer as a decimal or fraction, not as a percent.)

See Chapter 26 for more about fill-in questions.

Here's How to Crack It

The question asks for a probability based on data in a table. Probability is defined as $\frac{\text{\# of outcomes that give you what you want}}{\text{total \# of possible outcomes}}$. Read the table carefully to find the numbers to make the probability. There are 300 total students, so that is the *total \# of possible outcomes*. Of these 300 students, 30 are juniors taking culinary arts, so that is the *\# of outcomes that give you*

what you want. Therefore, the probability that a student chosen at random is a junior taking culinary arts is $\frac{30}{300}$. This answer cannot be entered into the fill-in box, which only accepts 5 characters when the answer is positive. All equivalent answers that fit will be accepted, so reduce the fraction or convert it to a decimal. The correct answer is $\frac{3}{30}$, $\frac{1}{10}$, 0.1, or another equivalent form.



RATES

Rate is a concept related to averages. Cars travel at an average speed. Work gets done at an average rate. Because the ideas are similar, the formulas you can use for rate problems are similar to the one for averages you learned about earlier. These formulas are $D = RT$ for distance and $W = RT$ for work.

Here's a simple example:

Problem: If a fisherman can tie 9 flies for fly fishing in an hour and a half, how long does it take him to tie one fly, in minutes?

Solution: First, convert the hour and a half to 90 minutes so your units are consistent. Then, fill in the formula with the work or amount done (9 flies) and the time (90 minutes).

$$W = RT$$

$$9 = (R)(90)$$

Divide 9 by 90 to get $R = \frac{1}{10}$, so the rate is one fly every 10 minutes.

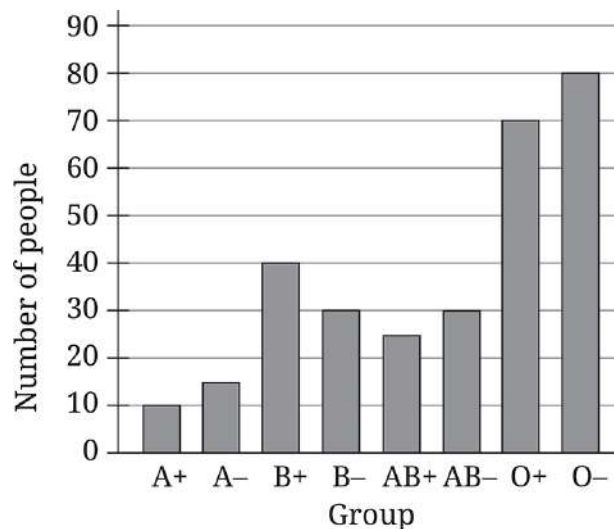
You have already seen that rate questions often involve proportions or unit conversion. As always, write down everything and label it carefully to avoid performing the wrong arithmetic operations.

IS THERE SCIENCE ON THE DIGITAL SAT?

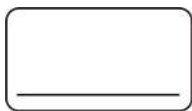
No, there isn't, but the test-writers like to make it look like there is. Some questions will use scientific topics and other "real-world" contexts to make things look more complicated. Don't worry about memorizing data about rock types or knowing what a lapping slurry is; just focus on the math.

Take a look at this next example.

 Mark for Review



18. A study about changes in blood type phenotypes after bone marrow transplants recorded the blood types of 300 people before they underwent a bone marrow transplant. How many more people in the study had blood type B- than blood type A+?



Here's How to Crack It

The question asks for the difference between two values based on a graph. Read the graph carefully and look up the necessary values. Find the bar on the graph for blood type B–, then look at the vertical axis to find the number of people with that blood type. The bar goes up to the line at 30, so 30 people have blood type B–. Do the same thing for A+ to see that 10 people have blood type A+. Translate *how many more* as subtraction to get $30 - 10 = 20$. The correct answer is 20.



You may also be asked to graph the data presented in a table. Your knowledge of positive and negative relationships will help—you can eliminate things with the wrong relationship.

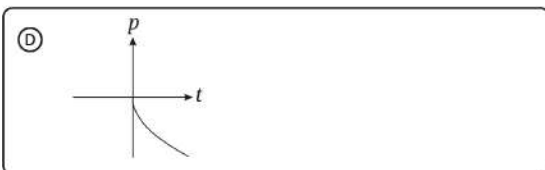
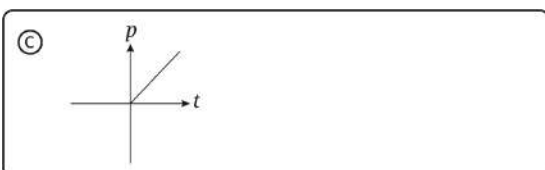
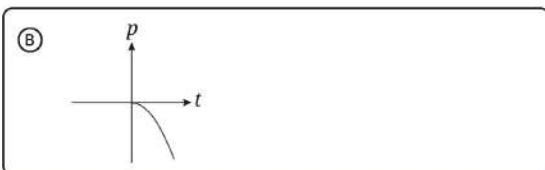
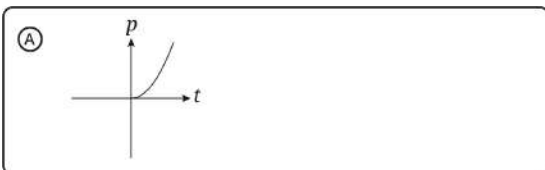


Mark for Review

Temperature in °C (<i>t</i>)	Vapor Pressure in kPa (<i>p</i>)
10	4
20	9
30	37
40	66
50	100

19. A student conducting experiments in class noticed that the temperature of a given liquid affected the vapor pressure of the liquid,

as shown in the table above. Which of the following graphs best represents the relationship between the temperature, t , and the vapor pressure, p , as indicated by the table?



Here's How to Crack It

The question asks for the graph that shows the relationship between temperature and vapor pressure. Notice that the vapor pressure increases as the temperature increases. The line or curve of best fit will go up as you follow the graph from left to right, so eliminate (B) and (D). To determine if the correct graph is (A) or (C), try roughly plotting the data points, and then look at your graph. Notice that the vapor pressure does not increase by the same number for each 10-degree temperature increase. This is an exponential increase, not a linear increase. Therefore, the graph will be curved. Eliminate (C). The correct answer is (A).



Advanced Arithmetic Drill

Work these questions using the advanced arithmetic techniques covered in this chapter. Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. The volume of a water tank decreased by a total of 144 liters over the course of 9 days. What was the rate, in liters per day, at which the volume decreased?

☐ A 16

☐ B 135

☐ C 153

☐ D 1,296

 Mark for Review

Number	Frequency
1	3
2	1
3	2
4	4

2. Which of the following lists of data is represented by the frequency table shown above?

☐ A 1, 1, 2, 2, 2, 3, 4, 4, 4, 4

☐ B 1, 1, 1, 2, 3, 3, 4, 4, 4, 4

☐ C 1, 2, 2, 3, 3, 3, 4, 4, 4, 4

☐ D 2, 3, 6, 16

 Mark for Review

3. Steve ran a 12-mile race at an average speed of 8 miles per hour. If Adam ran the same race at an average speed of 6 miles per hour, how many minutes longer did Adam take to complete the race than did Steve?

☐ A 12

☐ B 16

☐ C 24

☐ D 30

 Mark for Review

4. The amount of time that Amy walks is directly proportional to the distance that she walks. If she walks a distance of 2.5 miles in 50 minutes, how many miles will she walk in 2 hours?

☐ A 4.5

☐ B 5

☐ C 6

☐ D 6.5

 Mark for Review

5. A total of 140,000 votes was cast for two candidates, Candidate A and Candidate B. If Candidate A won by a ratio of 4 to 3, how many votes were cast for Candidate B?

☐ A 30,000

☐ B 40,000

☐ C 60,000

☐ D 80,000

 Mark for Review

6. A random sample of students at a local school was surveyed to determine the proportion of students with brown hair. Out of the 200 students sampled, 70 had brown hair. The margin of error of the survey was 6%. Which of the following is the most reasonable conclusion about the percentage of students with brown hair?

☐ A No less than 29% of the students have brown hair.

☐ B Between 29% and 41% of the students have brown hair.

☐ C Exactly 35% of the students have brown hair.

☐ D No more than 41% of the students have brown hair.

 Mark for Review

29, 32, 31, 29, 24, 25, 26

7. The data set shown above contains 7 integer values. If an 8th integer between 20 and 50 is added to the data set, the new mean will be 2

more than the current mean. What is the value of the 8th integer added to the data set?

 Mark for Review

8. Of all the houses in a certain neighborhood, 80% have garages. Of those houses with garages, 60% have two-car garages. If there are 56 houses with garages that are not two-car garages, how many houses are there in the neighborhood?

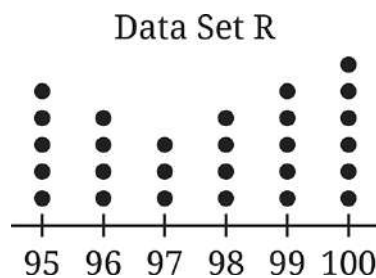
☐ A 93

☐ B 117

☐ C 156

☐ D 175

 Mark for Review



9. Data set R is represented by the dot plot above. Data set Q is created by subtracting 35 from each of the values in data set R. Which of the following statements correctly compares the standard deviations of data sets R and Q?

☐ A There is not enough information to compare these standard deviations.

☐ B The standard deviation of data set R is greater than the standard deviation of data set Q.

☐ C The standard deviation of data set R is less than the standard deviation of data set Q.

☐ D The standard deviation of data set R is equal to the standard deviation of data set Q.

 Mark for Review

10. On Tuesday, a watchmaker made 4 more watches than he made on Monday. If he made 16% more watches on Tuesday than on Monday, how many watches did he make on Tuesday?

☐ A 20

☐ B 21

☐ C 25

☐ D 29

ANSWERS TO FRACTIONS, DECIMALS, AND PERCENTS EXERCISE

1. $\frac{1}{2}$, 0.5, 50%

2. $\frac{3}{1}$, 3.0, 300%

3. $\frac{1}{200}$, 0.005, 0.5%

4. $\frac{1}{3}$, $0.333\overline{3}$, $33\frac{1}{3}\%$

ADVANCED ARITHMETIC DRILL ANSWERS AND EXPLANATIONS

1. A

The question asks for a rate. Begin by reading the question to find information about the rate. The question states that the volume *decreased by a total of 144 liters over the course of 9 days* and asks for the rate of decrease in *liters per day*. Divide the number of liters by the number of days to get $\frac{144 \text{ liters}}{9 \text{ days}} = \frac{16 \text{ liters}}{1 \text{ day}}$. The rate is 16 liters per day. The correct answer is (A).

2. B

The question asks for the list of numbers that correctly represents a frequency table. A frequency table has two columns: the left-hand column contains the values, and the right-hand column contains the number of times each value occurs, or its frequency. Work in bite-sized pieces and eliminate answer choices that do not match the data. According to the table, the number 1 has a frequency of 3, so the number 1 should be in the list 3 times. Eliminate (A), (C), and (D) because they do not have the number 1 in the list 3 times. Choice (B) shows the correct frequency for each value. The correct answer is (B).

3. D

The question asks for the difference in minutes between Steve's time and Adam's time. Use the formula $D = RT$ to calculate the time for each runner. Steve runs 12 miles at 8 miles per hour, so the formula becomes $12 = (8)(T)$. To find Steve's time, divide his distance by his rate, which means that he runs for $1\frac{1}{2}$ hours (or 1.5 if you're using your calculator). Adam runs the same 12 miles at 6 miles per hour, so the formula becomes $12 = (6)(T)$. This means that Adam runs for 2 hours. Adam takes one-half hour longer to complete the race, and one-half hour is 30 minutes. The correct answer is (D).

4. C

The question asks for the distance Amy can walk in two hours if she walks at a given rate. Since you know the time that Amy walked and the distance she walked are directly proportional, you can set up a proportion to show her distance $\div$ time. The time it took her to walk 2.5 miles is given in minutes and the requested time is in hours, so match the units in the proportion by putting $120(60 \times 2)$ minutes in the second half of the ratio: $\frac{2.5}{50} = \frac{x}{120}$. To solve, cross-multiply, and you'll get $50x = 2.5 \times 120$; $50x = 300$; $x = 6$ miles. The correct answer is (C).

5. C

The question asks for the number of votes Candidate B received. You know the ratio for the votes is 4 for Candidate A to every 3 for Candidate B, so the proportion of votes Candidate B received is 3 out of every 7 or $\frac{3}{7}$ of the total votes. There were 140,000 votes all together, so Candidate B received $\frac{3}{7} (140,000) = 60,000$ votes. The correct answer is (C).

6. **B**

The question asks for a reasonable conclusion based on survey results and a margin of error. Work in bite-sized pieces and eliminate after each piece. Start by determining the percent of surveyed students with brown hair by dividing the number of students with brown hair by the total number of students and multiplying by 100: $\frac{70}{200} \times 100 = 35\%$. A margin of error expresses the amount of random sampling error in a survey's results. Eliminate (C) because it is only the result, 35%, not a range above and below the result. The margin of error is 6%, meaning that results within a range of 6% above and 6% below the estimate are reasonable. The study found that 35% of students have brown hair, so the reasonable results for this study are between 29% and 41%. Eliminate (A) because it only addresses the lower limit. Keep (B) because it addresses both the lower limit and the upper limit.

Eliminate (D) because it only addresses the upper limit. The correct answer is (B).

7. **44**

The question asks for a value given information about the mean of a data set. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. Start by finding the average of the seven integers given in the question. There are 7 values, so $N = 7$. Find the *Total* by adding the seven integers to get $T = 29 + 32 + 31 + 29 + 24 + 25 + 26 = 196$. The average formula becomes $196 = (A)(7)$. Divide both sides of the equation by 7 to get $A = 28$. The question states that, after an eighth integer is added to the data set, *the new average will be 2 more than the current average*. Since the current average is 28, the new average will be $28 + 2 = 30$. There are now 8 integers in the data set, so plug $N = 8$ and $A = 30$ into the data set to get $T = (30)(8) = 240$. The question asks for the value of the eighth integer, so subtract the *Total* of the 7 integers from the *Total* of the 8 integers to get $240 - 196 = 44$ as the value of the eighth integer. The correct answer is 44.

8. **D**

The question asks for the number of houses in the neighborhood. Start by figuring out what percent of the houses do not have two-car garages. Since 60% of the houses with garages have two-car garages, 40% of the houses with garages do not have two-car garages. In other words, 40% of 80% of the houses do not have two-car garages.

Translate that into math to get $\frac{40}{100} \times \frac{80}{100} = 0.32$, or 32% of the houses.

The question states that 56 houses do not have two-car garages, which means 32% of the houses equals 56. Translating into math gives $\frac{32}{100} \times x = 56$. Solve for x , and you'll get 175. Another great option for questions that ask for something specific like number of houses is plugging in the answers. Either way, the correct answer is (D).

9. **D**

The question asks for a comparison of the standard deviations of two data sets. Standard deviation is a measure of the spread of a group of numbers. A group of numbers close together has a small standard deviation, whereas a group of numbers spread out has a large standard deviation. Subtracting 35 from each number will change the values in the data set but will not change how they are distributed. The dot plot of data set Q will look the same as the dot plot of data set R except for the values along the bottom of the dot plot. Because the shape of the dot plot remains the same, so does the standard deviation, and the two data sets have the same standard deviation. The correct answer is (D).

10. **D**

The question asks for the number of watches a watchmaker made on Tuesday. This is a specific amount, and there are numbers in the answer choices, so plug in the answers. Start with (B). If the watchmaker made 21 watches on Tuesday, then he must have made 17 watches on Monday. You know that he should have made 16% more watches on Tuesday than on Monday, so use the percent change

formula $\left(\frac{\text{difference}}{\text{original}} \times 100 \right)$ to see if you get 16%: $\frac{4}{17} \times 100 = 23.5\%$,

which is too big. Eliminate (B). You want the 4 watches to be a smaller percent of the total, so you need a bigger total. Try a bigger answer choice, like (D). If he made 29 watches on Tuesday, then he made 25 watches on Monday. Now the percent change is

$\frac{4}{25} = 0.16 = 16\%$ which is exactly what you want. The correct answer is (D).

Summary

- A ratio can be expressed as a fraction, but ratios are not fractions. A ratio compares parts to parts; a fraction compares a part to the whole.
- Set up proportions in the form $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.
- A percentage is just a convenient way of expressing a fraction with a denominator of 100.
- To convert a percentage to a fraction, put the percentage over 100 and reduce.
- To convert a fraction to a percentage, use your calculator to divide the top of the fraction by the bottom of the fraction. Then multiply the result by 100.
- To convert a percentage to a decimal, move the decimal point two places to the left. To convert a decimal to a percentage, move the decimal point two places to the right.
- In questions that require you to find a series of percentage increases or decreases, remember that each successive increase or decrease is performed on the result of the previous one.
- If you need to find the percent increase or decrease, use $\% \text{ change} = \frac{\text{difference}}{\text{original}} \times 100$.
- To find the mean, or average, of several values, add up the values and divide the total by the number of values.
- Use the formula $T = AN$ to solve questions involving means, or averages. The key to most average questions is finding the total.

- The median of a group of numbers is the number that is exactly in the middle of the group when the group is arranged from smallest to largest, as on a number line. If there is an even number of numbers, the median is the average of the two middle numbers.
- The mode of a group of numbers is the number in the group that appears most often.
- The range of a group of numbers is the difference between the greatest number in the group and the least number.
- Standard deviation is a measure of the spread, or distribution, of a group of numbers. More spread out means a large standard deviation, and more clustered together means a small standard deviation.
- Margin of error is the range above and below the value predicted by a survey within which the actual value is likely to be.
- Probability is expressed as a fraction:
 - Probability = $\frac{\text{number of outcomes that give you what you want}}{\text{total number of possible outcomes}}$
- On questions about rates, use the formulas $D = RT$ and $W = RT$. Be careful with the units—the Digital SAT will often require you to do a unit conversion such as minutes to hours or inches to feet.

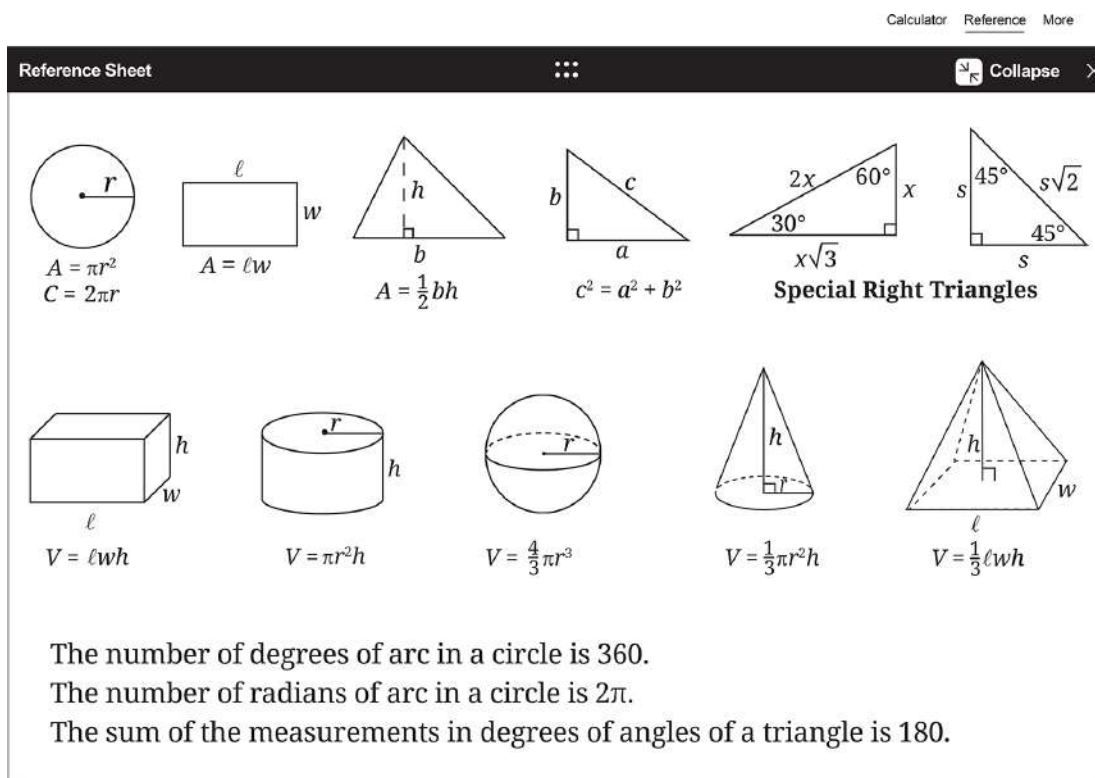
Chapter 25

Geometry and Trigonometry

The final math topics that are tested regularly on the Digital SAT are geometry and trigonometry. There will be 5–7 questions total on these topics, split between the two modules. These questions cover topics such as lines and angles, triangles, circles, and trig functions. This chapter covers each of those topics and more, and provides a step-by-step walkthrough for each type of question.

GEOMETRY ON THE DIGITAL SAT

Several geometry questions will test your knowledge of basic geometry rules, including formulas. Never fear! There is a reference sheet in the testing app that you can open at any time by clicking on the word Reference in the upper right corner of the screen. It saves time to know these facts and formulas ahead of time, but use the reference sheet to check or to look things up that you don't already know.



This reference sheet contains *some* of what you'll need to tackle geometry on the Digital SAT. In this chapter, we'll cover a basic approach for geometry questions and other information you'll need to know to handle geometry and trig questions on the Digital SAT.

Geometry: Basic Approach

For the handful of geometry questions that appear on the Digital SAT, we recommend the following step-by-step approach:

1. **Draw a figure** on your scratch paper. If the question contains the note “Figure not drawn to scale,” draw the figure using information in the question.
2. **Label the figure** with any information given in the question. Sometimes you can plug in for parts of the figure as well.
3. **Write down formulas** that you might need for the question.
4. **Ballpark** if you’re stuck or running short on time.

These four steps, combined with the techniques you’ve learned in the rest of this book and the geometry concepts this chapter will cover, will enable you to tackle any geometry question you might run across on the Digital SAT.

Before we dive in to the nitty-gritty, let’s try a question using this approach.

 Mark for Review

1. In triangle ABC , angle $B = 60^\circ$ and AC is perpendicular to BC . If $AB = x$, what is the area of triangle ABC , in terms of x ?

☐ $\frac{x^2\sqrt{3}}{8}$

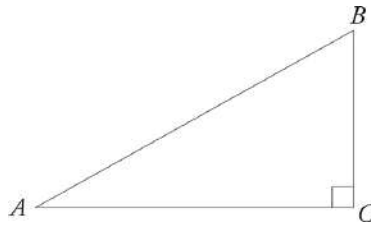
☐ $\frac{x^2\sqrt{3}}{4}$

☐ $\frac{x^2\sqrt{3}}{2}$

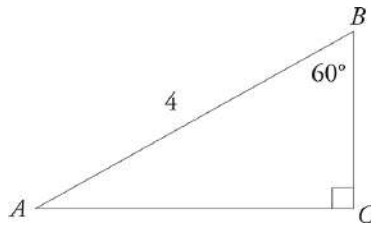
☐ $x^2\sqrt{3}$

Here's How to Crack It

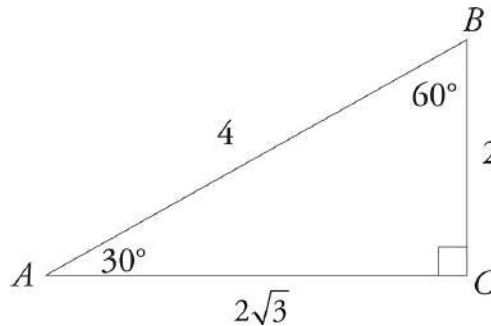
The question asks for the area of the triangle. Follow the steps outlined on the previous page. Start by drawing the figure. If AC is perpendicular to BC , then triangle ABC is a right triangle with the right angle at point C :



The next step is to label what you know. Angle $B = 60^\circ$ can go right into the figure. Because $AB = x$, you can plug in for x ; make $x = 4$. Label this information in the figure:



Next, figure out what other information you know. Because there are 180° in a triangle, angle $A = 180 - 90 - 60 = 30^\circ$. This is a 30° - 60° - 90° special right triangle, which you are given information about in the reference sheet. Based on the figure given in the box, the hypotenuse is equal to $2x$. (Note that this is a different x from the one you plugged in for; the test-writers are trying to confuse you.) So, if the hypotenuse is 4, $x = \frac{4}{2} = 2$; this is the side opposite the 30° angle, BC . The remaining side, AC , is $x\sqrt{3}$, which is $2\sqrt{3}$. Label this information in your figure:



Now write down the formula you need. The question is asking for the area, so use the area of a triangle formula from the reference sheet: $A = \frac{1}{2}bh$. Fill in what you know. Because this is a right triangle, you can use the two legs of the triangle as the base and the height. Make $b = 2\sqrt{3}$ and $h = 2$ in the equation and solve: $A = \frac{1}{2}(2\sqrt{3})(2) = 2\sqrt{3}$. This is your target; circle it. Now plug in $x = 4$ (that's the x from the question, NOT the x from the information in the reference sheet!) into each answer choice and eliminate what doesn't equal $2\sqrt{3}$. Only (A) works, so the correct answer is (A).



Now that we've covered how to approach geometry questions, let's look more closely at some of the geometry concepts you'll need for these questions.

LINES AND ANGLES

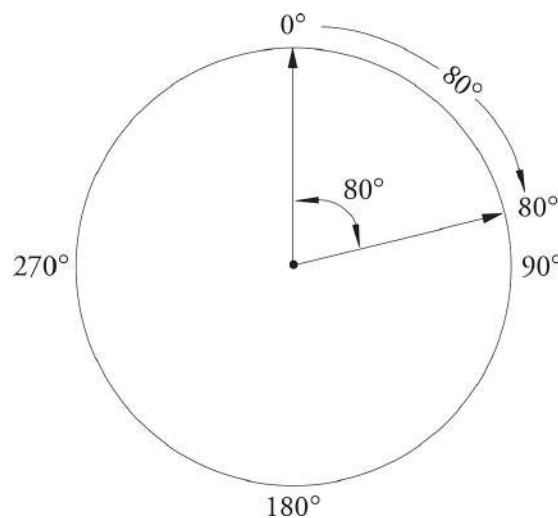
Here are the basic rules you need to know for questions about lines and angles on the Digital SAT.

1. A circle contains 360 degrees.

Every circle contains 360 degrees. Each degree is $\frac{1}{360}$ of the total distance around the outside of the circle. It doesn't matter whether the circle is large or small; it still has exactly 360 degrees.

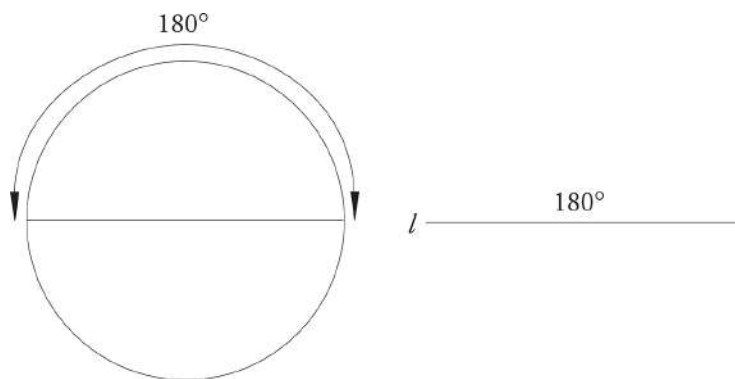
2. When you think about angles, remember circles.

An angle is formed when two line segments extend from a common point. If you think of the point as the center of a circle, the measure of the angle is the number of degrees enclosed by the lines when they pass through the edge of the circle. Once again, the size of the circle doesn't matter; neither does the length of the lines. Refer to the following figure.



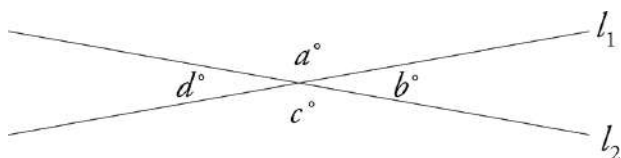
3. A line is a 180° angle.

You probably don't think of a line as an angle, but it is one. Think of it as a flat angle. The following drawings should help:



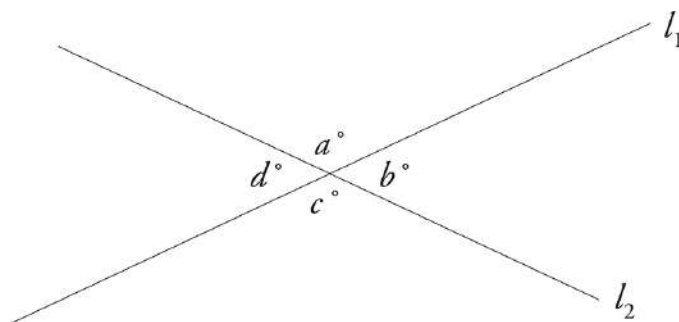
4. When two lines intersect, four angles are formed.

The following figure should make this clear. The four angles are indicated by letters.



5. When two lines intersect, the angles opposite each other will have the same measures.

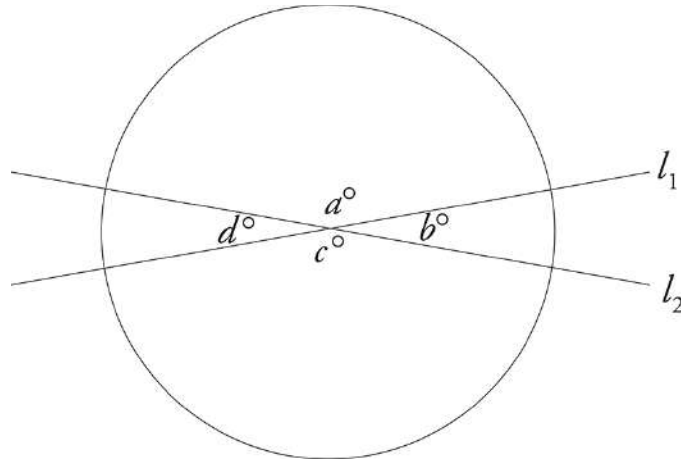
Such angles are called **vertical angles**. In the following figure, angles a and c are equal; so are angles b and d .



$$a + b + c + d = 360^\circ$$

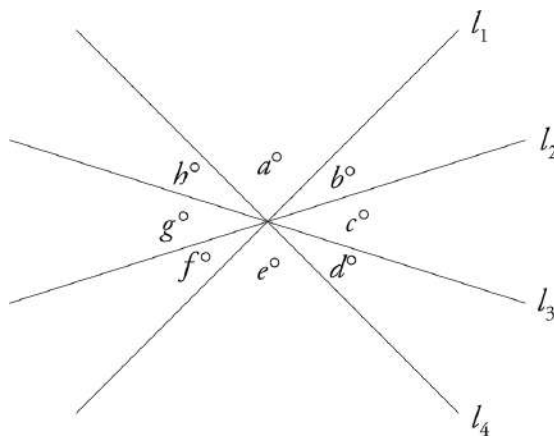
$$a = c, b = d$$

The measures of these four angles add up to 360° . (Remember the circle.)



$$a + b + c + d = 360^\circ$$

It doesn't matter how many lines you intersect through a single point. The total measure of all the angles formed will still be 360° .



$$a + b + c + d + e + f + g + h = 360^\circ$$

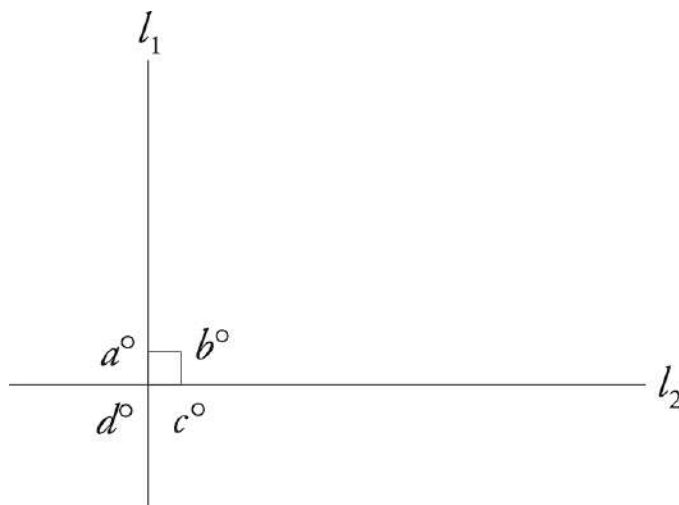
$$a = e, b = f, c = g, d = h$$

6. If two lines are perpendicular to each other, each of the four angles formed is 90° .

A 90° angle is called a ***right angle***.

Perpendicular:

Meeting at right (90°) angles



Angles a , b , c , and d all equal 90° .

The little box at the intersection of the two lines is the symbol for a right angle. If the lines are not perpendicular to each other, then none of the angles will be right angles. Don't assume that an angle is a right angle unless you are specifically told that it is a right angle, either in the question or with the right angle symbol.

Flip and Negate

If two lines are perpendicular, then their slopes are negative reciprocals; i.e., if l_1 has a slope of 2 and l_2 is perpendicular to l_1 , then l_2 must have a slope of $-\frac{1}{2}$.

7. When two parallel lines are cut by a third line, all of the small angles are equal, all of the big angles are equal, and the sum of any big angle and any small angle is 180° .

Parallel lines are two lines that never intersect, and the rules about parallel lines are usually taught in school with lots of big words. But we like to avoid big words whenever possible.

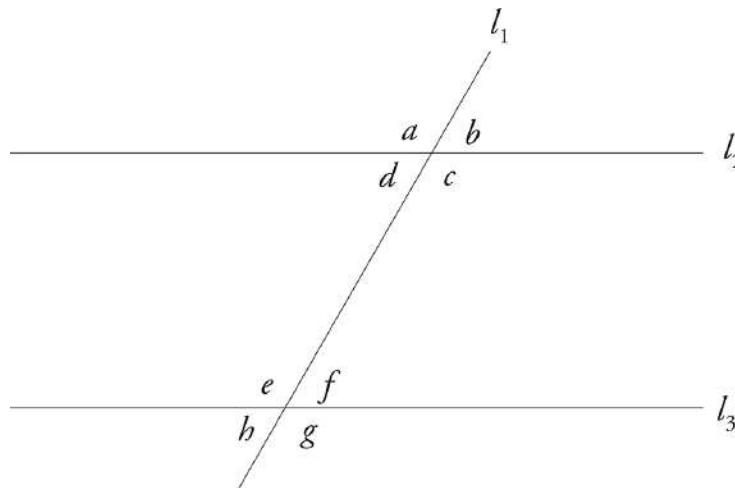
Simply put, when a line cuts through two parallel lines, two

kinds of angles are created: big angles and small angles. You can tell which angles are big and which are small just by looking at them. All the big angles look equal, and they are. The same is true of the small angles. Lastly, any big angle plus any small angle always equals 180° . (The test-writers like rules about angles that add up to 180° or 360° .)

Parallel Lines

Parallel lines have the same slope.

In any geometry question, never assume that two lines are parallel unless the question or figure specifically tells you so. The two lines in the following figure are parallel. Angle a is a big angle, and it has the same measure as angles c , e , and g , which are also big angles. Angle b is a small angle, and it has the same measure as angles d , f , and h , which are also small angles.



l_2 is parallel to l_3

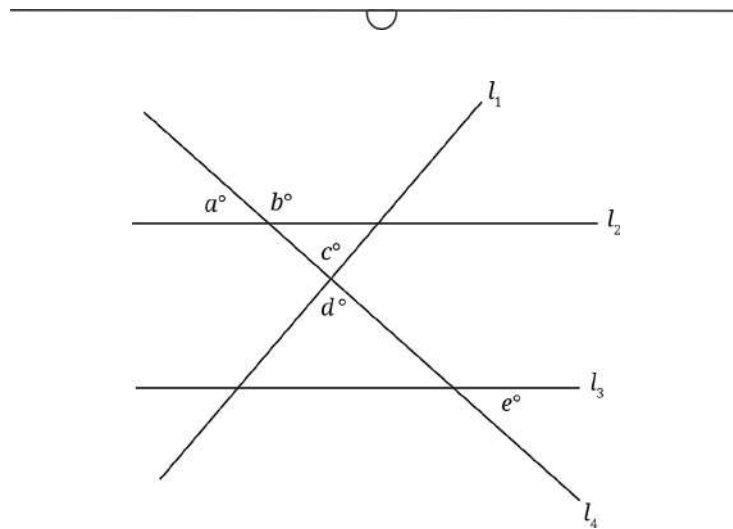
$$a = c = e = g$$

$$b = d = f = h$$

You should be able to see that the degree measures of angles a , b , c , and d add up to 360° . So do those of angles e , f , g , and h . If you have

trouble seeing it, draw a circle around the angles. What is the degree measure of a circle? Also, the sum of any small angle (such as d) and any big angle (such as g) is 180° .

Let's see how these concepts might be tested on the Digital SAT.



Note: Figure not drawn to scale.

 Mark for Review

2. In the figure above, l_2 is parallel to l_3 . If $b = 130$, what is the value of e ?

☐ A 40

☐ B 50

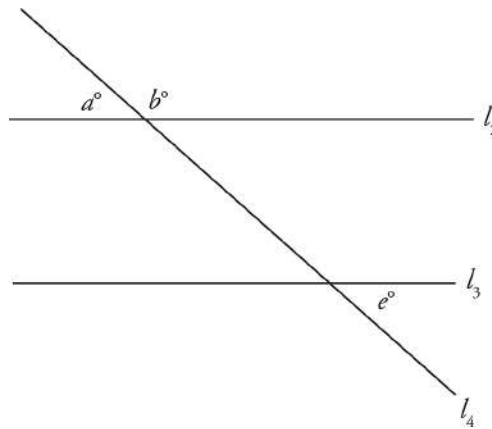
☐ C 90

☐ D 130

Here's How to Crack It

The question asks for the value of an angle on a figure. Use the Geometry Basic Approach. Start by redrawing the figure on your scratch paper. This figure has a lot going on, so only draw the parts you need and add more later if necessary. Angle e is between lines 3 and 4, and line 2 is parallel to line 3. That all seems important, so include it in your figure.

The drawing should look something like this:



Next, label the figure with information given in the question, and label angle b as 130° . When a geometry question states that two lines are parallel, it's almost certainly testing the rules about angles. Recall that when two parallel lines are cut by a third line, two kinds of angles are created: big and small. All of the small angles are equal to each other, all of the big angles are equal to each other, and any small angle + any big angle = 180° . Angle b is a big angle and angle e is a small angle, so $b + e = 180$. Plug in the measure of angle b to get $130 + e = 180$. Subtract 130 from both sides of the equation to get $e = 50$. The correct answer is (B).

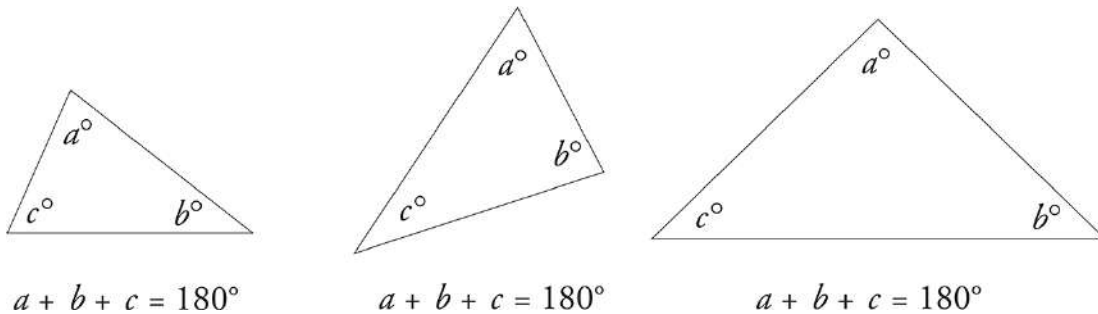


TRIANGLES

Here are some basic triangle rules you'll need to know for the Digital SAT.

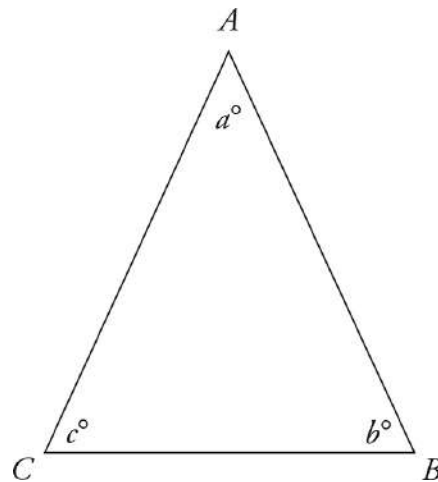
1. Every triangle contains 180° .

The word *triangle* means “three angles,” and every triangle contains three interior angles. The measure of these three angles always adds up to exactly 180° . You don’t need to know why this is true or how to prove it. You just need to know it. And we mean *know* it.



2. An isosceles triangle is one in which two of the sides are equal in length.

The angles opposite those equal sides are also equal because angles opposite equal sides are also equal.



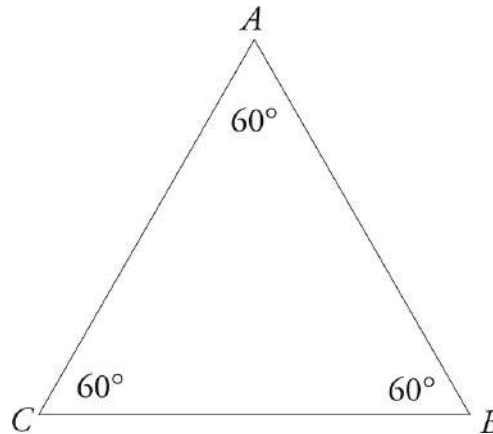
$$AB = AC \quad AB \neq BC$$

$$c = b \quad c \neq a$$

3. An equilateral triangle is one in which all three sides are equal in length.

Because the angles opposite equal sides are also equal, all three angles in an equilateral triangle are equal too. (Their measures are always 60°)

each.)



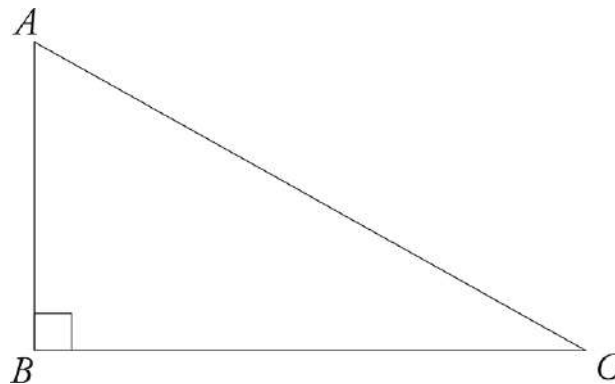
$$AB = BC = AC$$

Equilateral Triangles

An equilateral triangle is also isosceles.

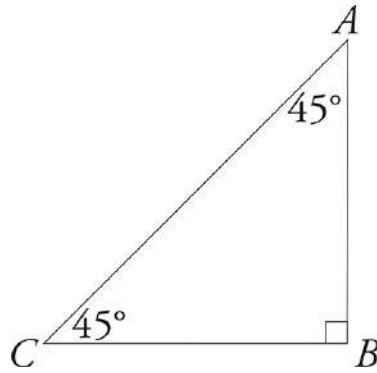
- 4. A right triangle is a triangle in which one of the angles is a right angle (90°).**

The longest side of a right triangle, which is always opposite the 90° angle, is called the ***hypotenuse***. The other two sides are called ***legs***.



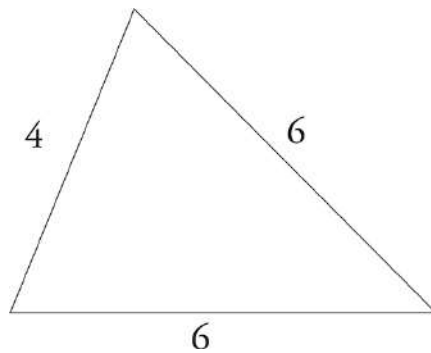
AC is the hypotenuse.

Some right triangles are also ***isosceles***. The angles in an isosceles right triangle always measure 45° , 45° , and 90° .



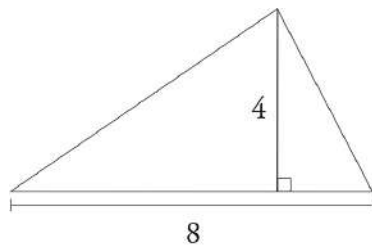
$$AB = BC$$

5. The perimeter of a triangle is the sum of the lengths of its sides.

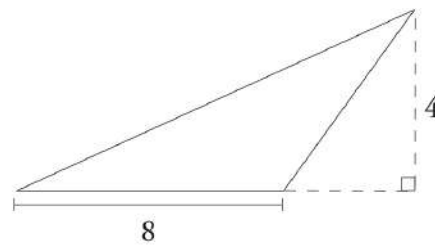


$$\text{perimeter} = 4 + 6 + 6 = 16$$

6. The area of a triangle is $\frac{1}{2}$ (base $\times$ height).



$$\text{area} = \frac{1}{2} (8 \times 4) = 16$$



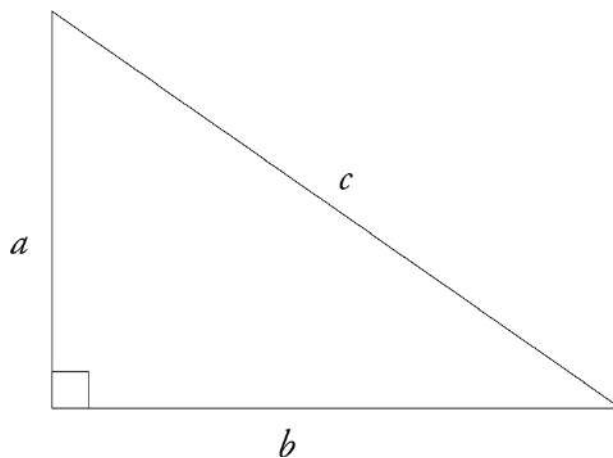
$$\text{area} = \frac{1}{2} (8 \times 4) = 16$$

In or Out

The height can be found with a line dropped inside or outside the triangle—just as long as it's perpendicular to the base.

Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides. As mentioned earlier, the hypotenuse is the longest side of a right triangle; it's the side opposite the right angle. The square of the hypotenuse is its length squared. Applying the Pythagorean Theorem to the following drawing, we find that $a^2 + b^2 = c^2$.

**Pythagorean Theorem**

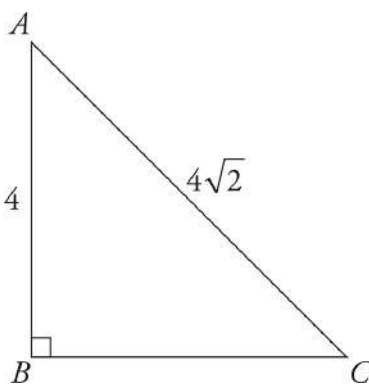
$a^2 + b^2 = c^2$, where c is the hypotenuse of a right triangle. Learn it; love it.

If you forget the Pythagorean Theorem, you can always look it up on the reference sheet.

3. In triangle ABC , angle B is a right angle, $AB = 4$, and $AC = 4\sqrt{2}$. If the perimeter of the triangle can be written in the form $4(\sqrt{2} + a)$, and a is a constant, what is the value of a ?

Here's How to Crack It

The question asks for the value of a constant given information about a triangle. Use the Geometry Basic Approach. Start by drawing a right triangle on your scratch paper and put the right angle symbol at B . Next, label the figure with information from the question. Label side AB as 4 and side AC as $4\sqrt{2}$. Notice that side AC is opposite the right angle, so it is the hypotenuse. The drawing should look something like this:



To find the length of the third side, use the Pythagorean Theorem: $a^2 + b^2 = c^2$. Plug in the known values to get $4^2 + b^2 = (4\sqrt{2})^2$. Square the numbers to get $16 + b^2 = 32$, then subtract 16 from both sides of the equation to get $b^2 =$

16. Take the square root of both sides of the equation to get $b = 4$. Label side BC as 4. This happens to be an isosceles right triangle, which is one of the triangles on the reference sheet. If you catch that, you might be able to find the length of the third side faster, but using the Pythagorean Theorem works well, too.

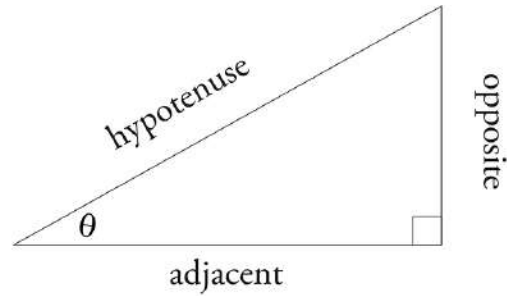
The perimeter of a geometric shape is the sum of the lengths of the sides, so $P = 4 + 4 + 4\sqrt{2}$, or $P = 8 + 4\sqrt{2}$. Set this equal to the different form of the perimeter given in the question to get $8 + 4\sqrt{2} = 4(\sqrt{2} + a)$. Distribute on the right side of the equation to get $8 + 4\sqrt{2} = 4\sqrt{2} + 4a$. Subtract $4\sqrt{2}$ from both sides of the equation to get $8 = 4a$. Divide both sides of the equation by 4 to get $2 = a$. The correct answer is 2.



TRIGONOMETRY

SOHCAHTOA

Trigonometry will appear on the Digital SAT Math section. But fear not! Many trigonometry questions you will see mostly require you to know the basic definitions of the three main trigonometric functions. **SOHCAHTOA** is a way to remember the three functions.



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

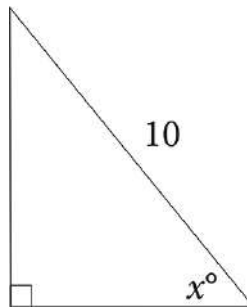
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Check out this next example.



 Mark for Review



4. In the triangle above, $\sin x = 0.8$ and $\cos x = 0.6$. What is the area of the triangle?

☐ A 0.48

☐ B 4.8

☐ C 24

☐ D 48

Here's How to Crack It

The question asks for the area of the triangle. Use the definitions of sine and cosine to find the two legs of the triangle. Sine is $\frac{\text{opposite}}{\text{hypotenuse}}$, so if $\sin x = 0.8$, then $0.8 = \frac{\text{opposite}}{10}$. Multiply both sides by 10 and you find the side opposite the angle with measure x° is 8. Similarly, cosine is $\frac{\text{adjacent}}{\text{hypotenuse}}$, so if $\cos x = 0.6$, then $0.6 = \frac{\text{adjacent}}{10}$. Multiply both sides by 10 to determine that the side adjacent to the angle with measure x° is 6. With those two sides, find the area. The formula for area is $A = \frac{1}{2}bh$, so $A = \frac{1}{2}(6)(8) = 24$. The correct answer is (C).



The test-writers love to ask you questions involving the Pythagorean Theorem along with SOHCAHTOA. See the following question.



 Mark for Review

5. In triangle ABC , AC is perpendicular to BC and $\cos(B) = \frac{12}{13}$. What is the value of $\tan(B)$?

☐ $\frac{5}{13}$

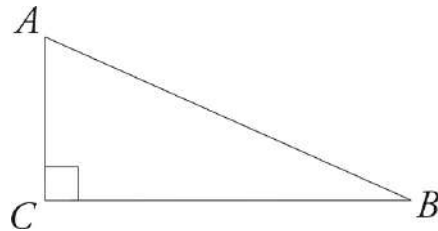
☐ $\frac{5}{12}$

☐ $\frac{12}{13}$

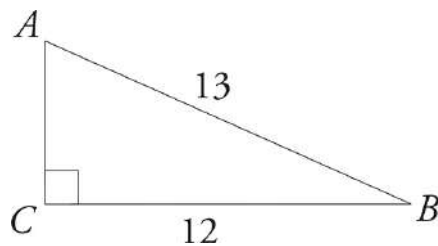
$\boxed{D} \frac{12}{5}$

Here's How to Crack It

The question asks for the value of the tangent of an angle. Use the Geometry Basic Approach: start by drawing triangle ABC .



Next, label what you can. You don't know the actual side lengths, but because $\cos (B) = \frac{12}{13}$, you do know the relationship between the side adjacent to angle B and the hypotenuse. You can plug in for this relationship: make BC (the side adjacent to the angle) 12 and AB (the hypotenuse) 13:



You need to find $\tan (B)$, which means you need $\frac{\text{opposite}}{\text{adjacent}}$. You already know the adjacent side is 12, but you still need the side opposite, AC . Use the Pythagorean Theorem to find the missing side:

$$a^2 + b^2 = c^2$$

$$12^2 + b^2 = 13^2$$

$$144 + b^2 = 169$$

$$b^2 = 25$$

$$b = 5$$

Therefore, $AC = 5$, and $\tan(B) = \frac{5}{12}$, so the correct answer is (B).

Your Friend the Rectangle

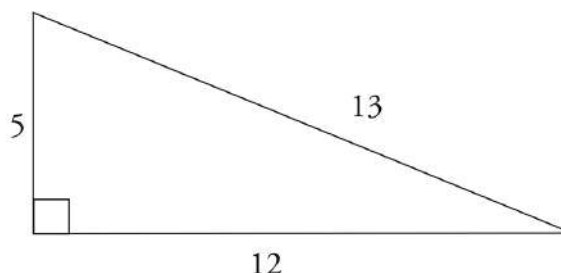
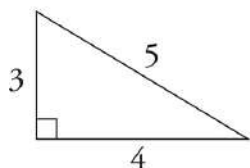
Be on the lookout for questions in which the application of the Pythagorean Theorem is not obvious. For example, every rectangle contains two right triangles. That means that if you know the length and width of the rectangle, you also know the length of the diagonal, which is the hypotenuse of both triangles.

Special Right Triangles

Both of the previous questions you worked also used special right triangles. While in the last question we used the Pythagorean Theorem to find the missing side, if you memorize these special triangles, you can avoid using the Pythagorean Theorem in a lot of cases.

When it comes to geometry questions involving right triangles, the Digital SAT is often fairly predictable, as questions tend to focus on certain relationships. In these questions, the triangles have particular ratios. There are two different types of special right triangles. The first involves the ratio of sides, and the second involves the ratio of angles.

The most common special right triangles with side ratios are known as **Pythagorean triples**. Here are the test-writers' favorites:



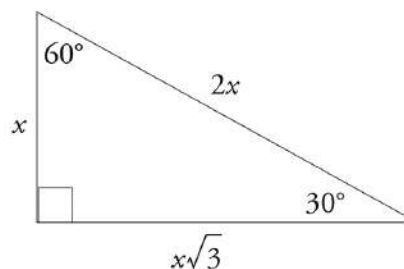
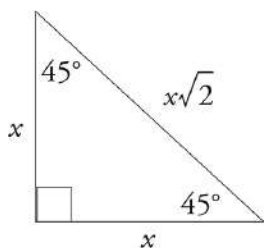
If you memorize these two sets of Pythagorean triples (3-4-5 and 5-12-13), you'll often be able to find the answer without using the Pythagorean Theorem. If you're given a right triangle with a side of 3 and a hypotenuse of 5, you know right away that the other side has to be 4. Likewise, if you see a right triangle with sides of 5 and 12, you know the hypotenuse must be 13.

Relax; It's Just a Ratio

A 3-4-5 triangle may be hiding, disguised as 6-8-10 or 18-24-30. It's all the same ratio, though, so be on the lookout.

The test-writers also like to use right triangles with sides that are simply multiples of the common Pythagorean triples. For example, you might see a 6-8-10 or a 10-24-26 triangle. These sides are simply the sides of the 3-4-5 and 5-12-13 triangles multiplied by 2.

There are two types of special right triangles that have a specific ratio of angles. They are the **30°-60°-90° triangle** and the **45°-45°-90° triangle**. The sides of these triangles always have the same fixed ratio to each other. The ratios are as follows:



Don't Forget the Reference Sheet!

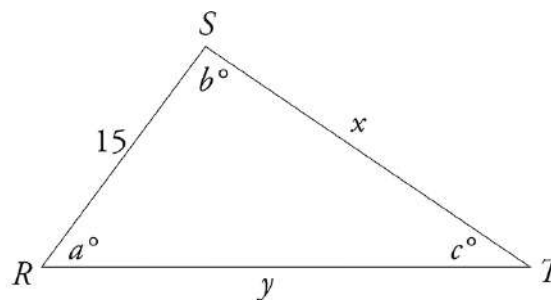
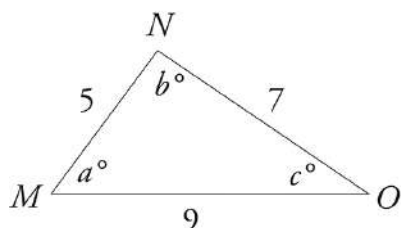
The relationships of the sides of special right triangles can be found in the reference sheet in the testing app, so you don't necessarily need to memorize them. However, you should be able to recognize them.

Let's talk about a 45° - 45° - 90° triangle first. Did you notice that this is also an isosceles right triangle? The legs will always be the same length. And the hypotenuse will always be the length of one leg times $\sqrt{2}$. Its ratio of side to side to hypotenuse is always $1:1:\sqrt{2}$. For example, if you have a 45° - 45° - 90° triangle with a leg length of 3, then the second leg length will also be 3 and the hypotenuse will be $3\sqrt{2}$.

Now let's talk about a 30° - 60° - 90° triangle. The ratio of shorter leg to longer leg to hypotenuse is always $1:\sqrt{3}:2$. For example, if the shorter leg of a 30° - 60° - 90° triangle is 5, then the longer leg would be $5\sqrt{3}$ and the hypotenuse would be 10.

Similar and Congruent Triangles

Similar triangles have the same shape, but they are not necessarily the same size. Having the same shape means that the angles of the triangles are identical and that the corresponding sides have the same ratio. Look at the following two similar triangles:



These two triangles both have the same set of angles, but they aren't the same size. Whenever this is true, the sides of one triangle are proportional to those of the other. Notice that sides NO and ST are both opposite the angle that is a° . These are called corresponding sides, because they correspond to the same angle. So the lengths of $\overline{NO}$ and $\overline{ST}$ are proportional to each other. In order to figure out the lengths of the other sides, set up a proportion: $\frac{MN}{RS} = \frac{NO}{ST}$. Now fill in the information that you know: $\frac{5}{15} = \frac{7}{x}$. Cross-multiply and you find that $x = 21$. You could also figure out the length of y : $\frac{NO}{ST} = \frac{MO}{RT}$. Therefore, $\frac{7}{21} = \frac{9}{y}$, and $y = 27$. Whenever you have to deal with sides of similar triangles, just set up a proportion.

Once in a while, a Digital SAT question will ask about what information is necessary in determining that two triangles are similar. These questions usually focus on angles, but it's important to know how both angles and sides can play a role. For two triangles to be similar, you need one of these relationships:

Two triangles are similar when at least **one** of the following is true:

- All three angles of the triangles are congruent (AAA).
- Pairs of sides of the triangles are in proportion, and the angle in between those sides is congruent (SAS).
- All three sides of one triangle are in proportion to the corresponding three sides of the other triangle (SSS).

In the previous example, there was enough information about triangles *MNO* and *RST* to use the AAA rule but not the SAS rule or the SSS rule.

You might also be asked about the information needed to prove that two triangles are congruent. Here are the rules for that:

Two triangles are congruent when at least **one** of the following is true:

- All three sides are equal (SSS).
- Two pairs of angles and the side between them are equal (ASA).
- Two pairs of sides and the angle between them are equal (SAS).
- Two pairs of angles and a side that *isn't* between them are equal (AAS).

Similar or Congruent?

All congruent triangles are similar.

Not all similar triangles are congruent.

AAA proves that triangles are similar but not necessarily congruent.

That's a lot, isn't it? The good news is that these topics aren't tested very often, so if they aren't in your POOD, you'll be okay. Here's a question that will help you review these concepts.

 Mark for Review

6. In triangles PQR and WXY , angles Q and X each measure 101° and angles R and Y each measure 37° . This information is sufficient to prove which of the following?

I. Triangles PQR and WXY are similar.

II. Triangles PQR and WXY are congruent.

☐ A Neither I nor II

☐ B I only

☐ C II only

☐ D I and II

Here's How to Crack It

The question asks what can be proven about two triangles using the given information. Eliminate (C) immediately because it is impossible for a triangle to be congruent without also being similar. The information in the

question is about angles only, and at least one pair of corresponding sides must be the same length for triangles to be congruent. There is not sufficient information to determine whether the triangles are congruent; eliminate (D). Since all triangles have 180 degrees and the triangles have two equal angles, the third angles must also equal each other. All three angles are congruent, which matches the AAA rule for similar triangles, so the information is sufficient to prove that the triangles are similar; eliminate (A). The correct answer is (B).

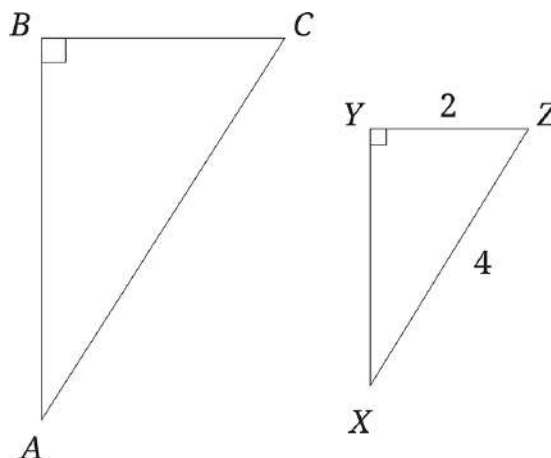


Similar Triangles and Trig

Finally, there's a special relationship between similar triangles and trigonometry. Side lengths in similar triangles are proportional, and the trigonometric functions give the proportions of the sides of a triangle. Therefore, if two triangles are similar, the corresponding trigonometric functions are equal! Let's look at how this might work in a question.



 Mark for Review



7. In the figure above, triangle ABC is similar to triangle XYZ . What is the value of $\cos(A)$?

☐ A $\frac{1}{2}$

☐ B $\frac{\sqrt{3}}{2}$

☐ C $\sqrt{3}$

☐ D 2

Here's How to Crack It

The question asks for the value of $\cos (A)$ but gives measurements on triangle XYZ . Because the two triangles are similar, the value of corresponding trigonometric functions will be equal. Therefore, $\cos (A) = \cos (X)$. The value of $\cos X$ is $\frac{\text{adjacent}}{\text{hypotenuse}}$ or $\frac{XY}{XZ}$. You could use the Pythagorean Theorem to find XY , but it's easier to use the special right triangle discussed earlier. Because the hypotenuse is twice one of the legs, you know this is a 30° - 60° - 90° triangle. YZ is the shortest side (x), so XY is $x\sqrt{3}$ or $2\sqrt{3}$. Therefore, $\cos (X) = \frac{2\sqrt{3}}{4}$, which reduces to $\frac{\sqrt{3}}{2}$. Because $\cos (X) = \cos (A)$, $\cos (A)$ also equals $\frac{\sqrt{3}}{2}$. The correct answer is (B).

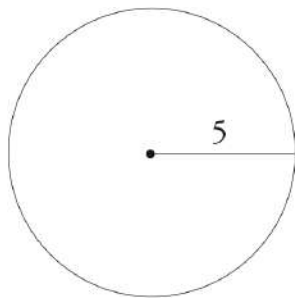


CIRCLES

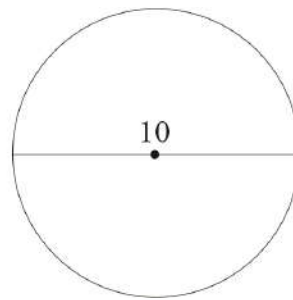
Here are the rules you'll need in order to tackle circle questions on the Digital SAT.

1. The circumference of a circle is $2\pi r$ or πd , where r is the radius of the circle and d is the diameter.

This information is in the reference sheet, so don't stress over memorizing these formulas. You will always be able to open the reference sheet in the testing app if you forget them. Just keep in mind that the diameter is always twice the length of the radius (and that the radius is half the diameter).



$$\text{circumference} = 2 \times \pi \times 5 = 10\pi$$



$$\text{circumference} = 10\pi$$

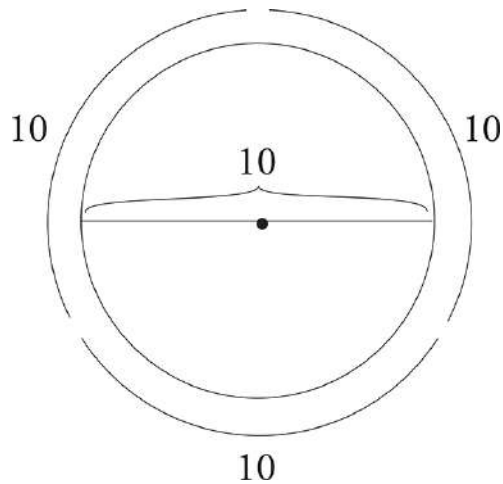
A Few Formulas

$$\text{Area} = \pi r^2$$

$$\text{Circumference} = 2\pi r \text{ or } \pi d$$

$$\text{Diameter} = 2r$$

In math class you probably learned that $\pi = 3.14$ (or even 3.14159). On the Digital SAT, $\pi = 3^+$ (a little more than 3) is a good enough approximation. Even with a calculator, using $\pi = 3$ will give you all the information you need to solve difficult Digital SAT multiple-choice geometry questions.

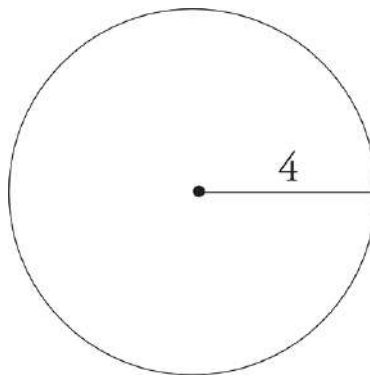


circumference = about 30

Leave That π Alone!

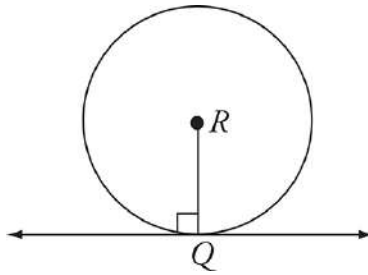
Most of the time, you won't multiply π out in circle questions. Because the answer choices will usually be in terms of π (6π instead of $18.849\dots$), you can save yourself some trouble by leaving your work in terms of π .

2. The area of a circle is πr^2 , where r is the radius of the circle.



$$\text{area} = \pi(4)^2 = 16\pi$$

3. A tangent is a line that touches a circle at exactly one point. A radius drawn from that tangent point forms a 90° angle with the tangent.



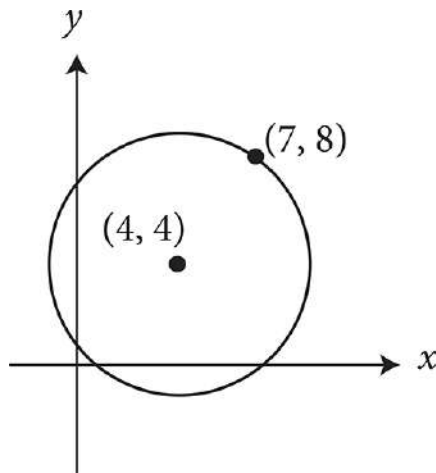
Circles Have Names?

If a question refers to Circle R, it means that the center of the circle is point R.

Let's see how these rules can show up on the Digital SAT.



 Mark for Review



8. The circle defined by the equation $(x - 4)^2 + (y - 4)^2 = 25$ has its center at point $(4, 4)$ and includes point $(7, 8)$ on the circle. This is shown in the figure above. What is the area of the circle shown?

☐ A 5π

☐ B 10π

☐ C 16π

☐ D 25π

Here's How to Crack It

The question asks for the area of the circle, so write down the formula for area of a circle: $A = \pi r^2$. That means you need to determine the radius of the circle. If you remember the circle formula from Chapter 23, you simply need to recall that $r^2 = 25$ and multiply by π to find the area. If not, you can find the distance between (4, 4) and (7, 8) by drawing a right triangle. The triangle is a 3-4-5 right triangle, so the distance between (4, 4) and (7, 8) (and thus the radius) is 5. If the radius is 5, then the area is $\pi(5)^2$, or 25π . The correct answer is (D).

Converting Degrees to Radians

Some geometry questions will ask you to convert an angle measurement from degrees to radians. While this may sound scary, doing this conversion only requires that you remember that $180^\circ = \pi$ radians. Use this relationship, which is included on the reference sheet, to set up a proportion and convert the units.

 Mark for Review

9. An angle in a circle measures $\frac{\pi}{6}$ radians. What is the measure of the angle in degrees?

Here's How to Crack It

The question asks for the measure of an angle in degrees. Use the relationship between radians and degrees to set up a proportion. If $180^\circ = \pi$ radians, the proportion will look like this:

$$\frac{180^\circ}{\pi \text{ radians}} = \frac{x^\circ}{\frac{\pi}{6} \text{ radians}}$$

Cross-multiply to get $(\pi)(x) = (180)\left(\frac{\pi}{6}\right)$. Simplify the right side of the equation to get $\pi x = 30\pi$, then divide both sides of the equation by π to get $x = 30$. The correct answer is 30.

30

Arcs and Sectors

Many circle questions on the Digital SAT will not ask about the whole circle. Rather, you'll be asked about arcs or sectors. Both arcs and sectors are portions of a circle: arcs are portions of the circumference, and sectors are portions of the area. Luckily, both arcs and sectors have the same relationship with the circle, based on the central angle (the angle at the center of the circle that creates the arc or sector):

$$\frac{\text{part}}{\text{whole}} = \frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{2\pi r} = \frac{\text{sector area}}{\pi r^2}$$

Note that these relationships are all proportions. Arcs and sectors are proportional to the circumference and area, respectively, as the central angle is to 360° .

Questions sometimes refer to “minor” or “major” arcs or sectors. A minor arc or sector is one that has a central angle of less than 180° , whereas a major arc or sector has a central angle greater than 180° (in other words, it goes the long way around the circle). Let’s see how arcs and sectors might show up in a question.



 Mark for Review

10. Points A and B lie on circle O (not shown). $AO = 3$ and angle $O = 120^\circ$. What is the area of minor sector AOB ?

☐ A $\frac{\pi}{3}$

☐ B π

☐ C 3π

☐ D 9π

Here’s How to Crack It

The question asks for the area of minor sector AOB in circle O . Because O is the name of the circle, it’s also the center of the circle, so AO is the radius. Angle O is the central angle of sector AOB , so you have all the pieces you need to find the sector. Put them into a proportion:

$$\frac{120^\circ}{360^\circ} = \frac{x}{\pi (3)^2}$$

Cross-multiply to get $360x = 1,080\pi$ (remember to not multiply out π). Divide both sides by 360, and you get $x = 3\pi$. The correct answer is (C).



Relationship Between Arc and Angle in Radians

Sometimes you'll be asked for an arc length, but you'll be given the angle in radians instead of degrees. Fear not! Rather than making the question more complicated, the test-writers have actually given you a gift! All you need to do is memorize this formula:

$$s = r\theta$$

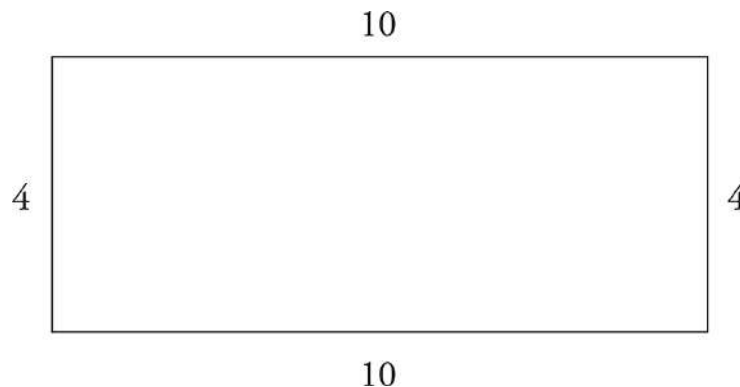
In this formula, s is the arc length, r is the radius, and θ is the central angle in radians. If you know this formula, these questions will be a snap!

When an arc measure is given in degrees, it's even simpler: the arc is the same number of degrees as the angle that defines it.

RECTANGLES AND SQUARES

Here are some rules you'll need to know about rectangles and squares:

- 1. The perimeter of a rectangle is the sum of the lengths of its sides.**
Just add them up.



$$\text{perimeter} = 10 + 4 + 10 + 4 = 28$$

Little Boxes

Here's a progression of quadrilaterals from least specific to most specific:

quadrilateral is any 4-sided figure

↓

parallelogram is a quadrilateral in which opposite sides are parallel

↓

rectangle is a parallelogram in which all angles = 90°

↓

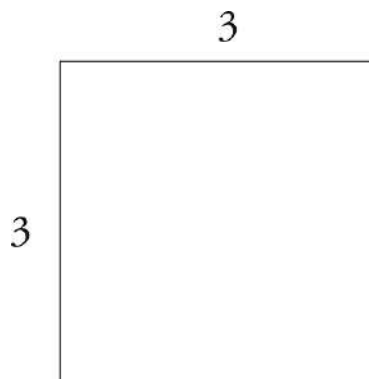
square is a rectangle in which all sides are equal

2. The area of a rectangle is length $\times$ width.

The area of the preceding rectangle, therefore, is 10×4 , or 40.

3. A square is a rectangle whose four sides are all equal in length.

The perimeter of a square, therefore, is four times the length of any side. The area is the length of any side squared.



$$\text{perimeter} = 4(3) = 12$$

$$\text{area} = 3^2 = 9$$

4. In rectangles and squares all angles are 90° angles.

It can't be a square or a rectangle unless all angles are 90° .

Let's check out an example.

 Mark for Review

11. If the perimeter of a square is 28, what is the length of the diagonal of the square?

☐ A $2\sqrt{14}$

☐ B $7\sqrt{2}$

☐ C $7\sqrt{3}$

☐ D 14

Here's How to Crack It

The question asks for the diagonal of a square based on its perimeter. The perimeter of a square is $4s$. So, $28 = 4s$. Divide by 4 to find $s = 7$. The diagonal of a square divides the square into two 45° - 45° - 90° triangles, with sides in the ratio of x : x : $x\sqrt{2}$. If the side is 7, the diagonal is $7\sqrt{2}$. The correct answer is (B).

VOLUME

Volume questions on the Digital SAT can seem intimidating at times. The test-writers love to give you questions featuring unusual shapes such as pyramids and spheres. Luckily, the reference sheet in the testing app contains all the formulas you will ever need for volume questions on the Digital SAT. Simply apply the Basic Approach for geometry using the given formulas and you'll be in good shape (pun entirely intended)!

Let's look at an example.

 Mark for Review

12. A sphere has a volume of 36π . What is the surface area of the sphere?
(The surface area of a sphere is given by the formula $A = 4\pi r^2$.)

☐ A 3π

☐ B 9π

☐ C 27π

☐ D 36π

Here's How to Crack It

The question asks for the surface area of a sphere given its volume. Start by writing down the formula for volume of a sphere from the reference sheet: $V = \frac{4}{3}\pi r^3$. Put what you know into the equation: $36\pi = \frac{4}{3}\pi r^3$. From this you can solve for r . Divide both sides by π to get $36 = \frac{4}{3}r^3$. Multiply both sides by 3 to clear the fraction: $36(3) = 4r^3$. Note we left 36 as 36, because the next step is to divide both sides by 4, and 36 divided by 4 is 9, so $9(3) = r^3$ or $27 = r^3$. Take the cube root of both sides to get $r = 3$. Now that you have the radius, use the formula provided to find the surface area: $A = 4\pi(3)^2$, which comes out to 36π . The correct answer is (D).

BALLPARKING

You may be thinking, “Wait a second, isn’t there an easier way?” By now, you should know that of course there is, and we’re going to show you. On many Digital SAT geometry questions, you won’t have to calculate an exact answer. Instead, you can estimate an answer choice. We call this ***Ballparking***, a strategy mentioned earlier in this book.

Ballparking is extremely useful on Digital SAT geometry questions. At the very least, it will help you avoid careless mistakes by immediately eliminating answers that could not possibly be correct. On many questions, Ballparking will allow you to find the answer without even working out the problem at all.

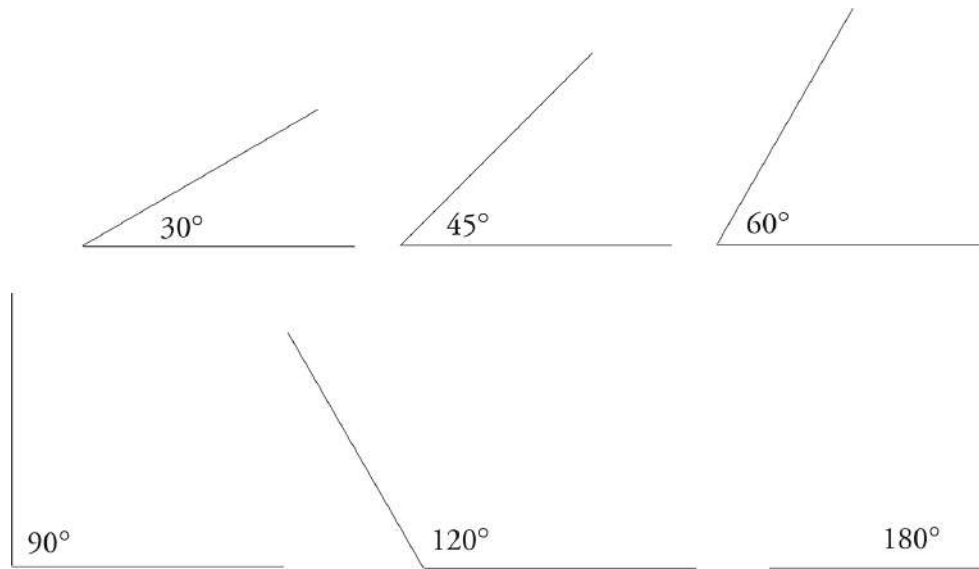
Rocket Science?

The SAT is a college admissions test, not an exercise in precision. Because approximately 33 of its 44 Math questions are multiple-choice, you can afford to approximate numbers like π , $\sqrt{2}$, and $\sqrt{3}$ (3+, 1.4, and 1.7+, respectively).

For example, on many Digital SAT geometry questions, you will be presented with a drawing in which some information is given and you will be asked to find some of the information that is missing. In most such questions, you’re expected to apply some formula or perform some calculation, often an algebraic one. But look at the drawing and make a rough estimate of the answer (based on the given information) before you try to work it out. You might be able to eliminate an answer choice or two or even narrow it down to one.

The basic principles you just learned (such as the number of degrees in a triangle and the fact that $\pi \approx 3$) will be enormously helpful to you in Ballparking on the Digital SAT.

Even though many geometric figures are marked with a note that they are not drawn to scale, you will also find it very helpful if you have a good sense of how large certain common angles are. Study the following examples.



How High Is the Ceiling?

If your friend stood next to a wall in your living room and asked you how high the ceiling was, what would you do? Would you get out your trigonometry textbook and try to triangulate using the shadow cast by your pal? Of course not. You'd look at your friend and think something like this: "Dave's about 6 feet tall. The ceiling's a couple of feet higher than he is. It must be about 8 feet high."

Your Ballpark answer wouldn't be exact, but it would be close. If someone later claimed that the ceiling in the living room was 15 feet high, you'd be able to tell her with confidence that she was mistaken.

You'll be able to do the same thing on the Digital SAT. If line segment A has a length of π and line segment B is exactly half as long, then the length of line segment B is a little more than 1.5. All such questions are ideal for Ballparking.

The Correct Choice

Remember that the Digital SAT is a mostly multiple-choice test. This means that you don't always have to come up with an answer; you just have to identify the correct one from among the four choices provided.

PLUGGING IN

As you learned already, Plugging In is a powerful technique for solving Digital SAT algebra questions. It is also very useful on geometry problems. For some questions, you will be able to plug in values for missing information and then use the results either to find the answer directly or to eliminate answers that cannot be correct.

Here's an example.

 Mark for Review

13. The base of triangle T is 40 percent less than the length of rectangle R . The height of triangle T is 50 percent greater than the width of rectangle R . The area of triangle T is what percent of the area of rectangle R ?

☐ A 10

☐ B 45

☐ C 90

☐ D 110

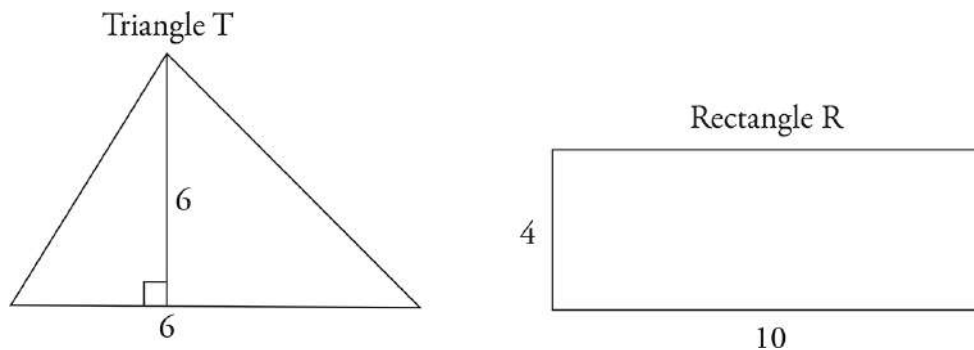
**Watch Us Crack It**

Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for the relationship between the areas of triangle T and rectangle R. This is a challenging question. Don't worry—you'll still be able to find the right answer by sketching and plugging in.

When plugging in, always use numbers that are easy to work with. Say the length of the rectangle is 10; that means that the base of the triangle, which is 40 percent smaller, is 6. If you plug 4 in for the width of rectangle R, the height of triangle T is 6. You should come up with two sketches that look like this:



T has an area of $\frac{1}{2}bh$, or 18. R has an area of 40. Now set up the translation:

$18 = \frac{x}{100}(40)$, where x represents what percent the triangle is of the rectangle.

Solve for x and you get 45. The correct answer is (B).



Drill 1: Geometry

Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. If a rectangular swimming pool has a volume of 16,500 cubic feet, a uniform depth of 10 feet, and a length of 75 feet, what is the width of the pool, in feet?

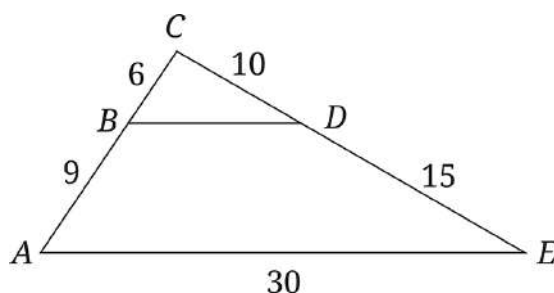
☐ A 22

☐ B 26

☐ C 32

☐ D 110

 Mark for Review



2. In the figure above, $\overline{AE}$ is parallel to $\overline{BD}$. what is the length of $\overline{BD}$?

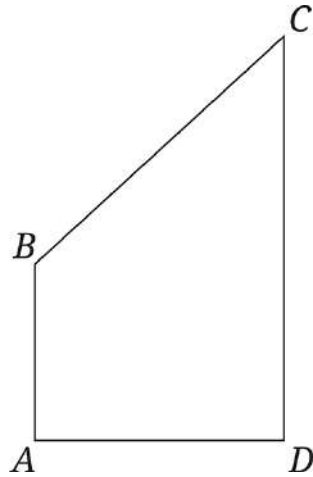
☐ A 8

☐ B 9

☐ C 12

☒ 15

 Mark for Review



3. In the quadrilateral above, the ratio of side length $\overline{AD}$ to $\overline{BC}$ is 5 to 7. If the length of $\overline{AD}$ is decreased by 1, how must the length of $\overline{BC}$ change to maintain this ratio?

☐ A It must decrease by 1 unit.

☐ B It must increase by 1 unit.

☐ C It must decrease by 1.4 units.

☐ D It must increase by 1.4 units.

 Mark for Review

4. Cube X has a side length 22 times the side length of cube Y. How many times greater is the volume of cube X than that of cube Y?

 Mark for Review

5. In right triangle ABC , angle A is a right angle, $AB = x$, and $BC = y$. The value of AC can be represented by which of the following expressions?

☐ A $\sqrt{y - x}$

☐ B $\sqrt{y^2 - x^2}$

☐ C $\sqrt{x^2 + y^2}$

☐ D $y - x$

 Mark for Review

6. A pyramid has a height of 4 centimeters (cm) and a regular hexagonal base with an area of 15 cm^2 . If the pyramid has a mass of 21.129 grams (g), what is the density of the pyramid in $\frac{\text{g}}{\text{cm}^3}$?

☐ A 1.06

☐ B 1.18

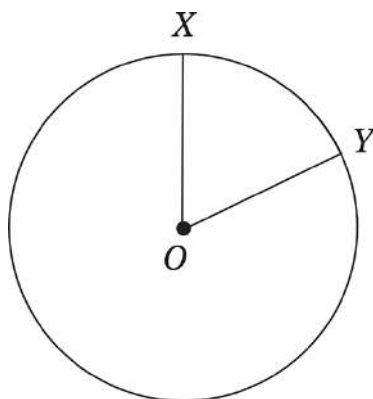
☐ C 2.09

☐ D 6.51

Drill 2: Trigonometry

Answers and explanations can be found on [this page](#).

 Mark for Review



1. In the figure above, circle O has a radius of 8, and angle XOY measures $\frac{5}{16}\pi$ radians. What is the measure of minor arc XY ?

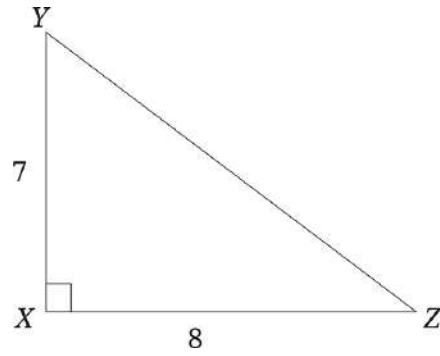
☐ A $\frac{5}{16}\pi$

☐ B $\frac{5}{2}\pi$

☐ C 5π

☐ D 16π

 Mark for Review



2. What is the value of $\tan(Z)$?

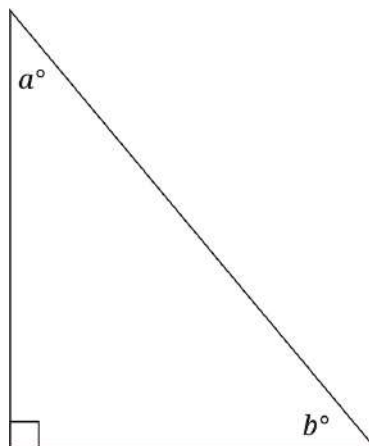
☐ A $\frac{7\sqrt{115}}{115}$

☐ B $\frac{8\sqrt{115}}{115}$

☐ C $\frac{7}{8}$

☐ D $\frac{8}{7}$

 Mark for Review



3. In the figure above, $\sin a = x$. What is the value of $\cos b$?

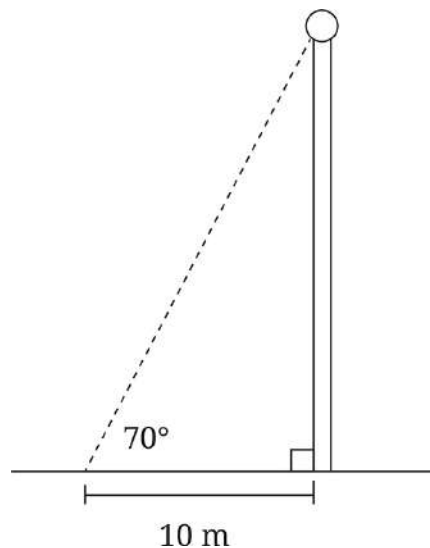
☐ A x

☐ B $\frac{1}{x}$

☐ C $|1 - x|$

☐ D $\frac{90 - x}{90}$

 Mark for Review



4. Martin wants to know how tall a certain flagpole is. Martin walks 10 meters from the flagpole, lies on the ground, and measures an angle of 70° from the ground to the base of the ball at the top of the flagpole. Approximately how tall, in meters, is the flagpole from the ground to the base of the ball at the top of the flagpole?

☐ A 3 meters

☐ B 9 meters

☐ C 27 meters

D 29 meters

DRILL 1 AND DRILL 2 ANSWERS AND EXPLANATIONS

Drill 1: Geometry

1. A

The question asks for the width of the pool. For this question, you need to know that volume equals *length* $\times$ *width* $\times$ *height*. You know that the volume is 16,500, the depth (or height) is 10, and the length is 75. Just put those numbers into the formula: $16,500 = 75 \times w \times 10$. Use your calculator to solve for w , which equals 22. The correct answer is (A).

2. C

The question asks for a value in a geometric figure. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper.

Split the figure into two triangles to make things clearer: triangle ACE and triangle BCD . Next, label the figure with the given information.

The only new information is that $\overline{AE}$ is parallel to $\overline{BD}$. Use the knowledge that parallel lines cut by a third line create big and small angles, and label angles A and B as congruent and angles D and E as congruent. Triangles ACE and BCD have the same angle C , which means they have three congruent angles and are similar triangles. The sides of similar triangles have an equivalent ratio, so set up a proportion. First, add CD to DE to get the length of CE : $10 + 15 = 25$.

Now use the ratio of CD to CE to find the ratio of BD to AE . The proportion is $\frac{10}{25} = \frac{x}{30}$. Cross-multiply to get $(25)(x) = (10)(30)$, which becomes $25x = 300$. Divide both sides of the equation by 25 to get $x = 12$. The correct answer is (C).

3. C

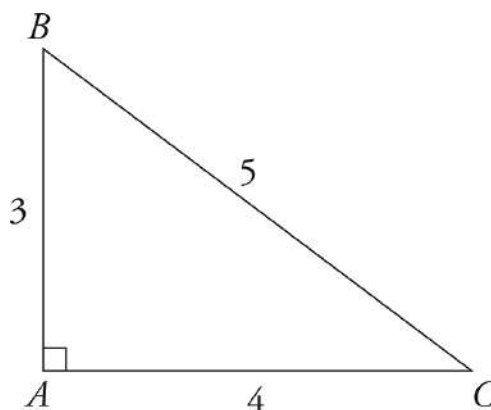
The question asks for the change in a value given a proportion. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper. Next, label the figure with information from the question. Since no specific numbers are given, only a ratio, plug in. Label AD as 5 and BC as 7. Draw the figure a second time and label it based on the change described in the question. Since AD is decreased by 1, label AD on the second figure as 4. Since AD decreased and the ratio stays the same, BC must also decrease. Eliminate (B) and (D) because they would both make BC longer. To find the new length of BC , set up a proportion for $\frac{AD}{BC}$: $\frac{5}{7} = \frac{4}{x}$. Cross-multiply to get $(7)(4) = (5)(x)$, which becomes $28 = 5x$. Divide both sides of the equation by 5 to get $x = 5.6$. Since the original length of BC was 7, the change is $7 - 5.6 = 1.4$. Eliminate (A) because it gives the wrong value for the decrease. The correct answer is (C).

4. 10648

The question asks for the relationship between the volumes of two geometric figures. Use the Geometry Basic Approach. Draw two cubes—or at least two squares to represent one face of each cube—on the scratch paper. Next, label the figure with the given information. No specific values are given, so plug in and label the side length of cube Y as 2. Since the side length of cube X is 22 times that of cube Y, label the side length of cube X as 44. The reference sheet doesn't give the formula for the volume of a cube, but it does give the volume of a rectangular solid: $V = lwh$. All three sides of a cube are the same length, so the formula becomes $V = s^3$. Plug in the two side lengths to determine the volume of each cube. The volume of cube Y becomes $V = 2^3$, or $V = 8$. The volume of cube X becomes $V = 44^3$, or $V = 85,184$. To answer the final question, divide the volume of cube X by the volume of cube Y to get $\frac{85,184}{8} = 10,648$. Leave out the comma when entering the answer in the fill-in box. The correct answer is 10648.

5. B

The question asks for an expression that represents a value in a geometric figure. Use the Geometry Basic Approach. Start by drawing a triangle on the scratch paper with a right angle at A. Next, label the figure with the information given. No specific values are given, so plug in. To make things easier, use one of the Pythagorean triples and make it a 3-4-5 right triangle. The drawing should look something like this:



Write down $x = 3$ and $y = 5$ to keep track of which variable has which value. The question asks for AC , which is 4. This is the target value; write it down and circle it. Now plug $x = 3$ and $y = 5$ into the answer choices and eliminate any that do not match the target value. Choice (A) becomes $\sqrt{5-3} = \sqrt{2}$. This does not match the target value of 4, so eliminate (A). Choice (B) becomes $\sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$. This matches the target, so keep (B), but check the remaining answers just in case. Choice (C) becomes $\sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{36} = 6$; eliminate (C). Choice (D) becomes $5 - 3 = 2$; eliminate (D). The correct answer is (B).

6. A

The question asks for the density of a geometric figure. Start by determining the volume of the pyramid. Look up the formula for the volume of a right regular pyramid in the reference sheet and write it down: $V = \frac{1}{3}lwh$. The base of a pyramid is the length times the width—or the area of the base—and the area of the base is given as 15 cm^2 , so plug in 15 cm^2 for lw . Also plug in 4 cm for h . The formula becomes $V = \frac{1}{3}(15 \text{ cm}^2)(4 \text{ cm})$, or $V = 20 \text{ cm}^3$. Next, use the units to

figure out how to solve for the density. Since density is in $\frac{\text{g}}{\text{cm}^3}$, the mass is in g, and the volume is in cm^3 , density must be mass divided by volume. This happens to be the formula for density, which can be written as $D = \frac{m}{V}$. Plug in $m = 21.129$ and $V = 20$ to get $D = \frac{21.129}{20}$, or $D = 1.05645$. This rounds to 1.06, which is an answer choice. The correct answer is (A).

Drill 2: Trigonometry

1. B

The question asks for the measure of minor arc XY . Since the question gives you the measure of the central angle in radians, you can use the formula $s = r\theta$ to find the arc length: $s = (8)\left(\frac{5}{16}\pi\right) = \frac{40}{16}\pi$, which reduces to $\frac{5}{2}\pi$. The correct answer is (B).

2. C

The question asks for the tangent of an angle, which is defined as $\frac{\text{opposite}}{\text{adjacent}}$. The side opposite angle Z is 7, and the side adjacent to this angle Z is 8, so $\tan(Z) = \frac{7}{8}$. The correct answer is (C).

3. A

The question asks for the value of $\cos b$. You can plug in when you're dealing with a geometry question with unknowns. When you're

plugging in for a right triangle, use one of the special right triangles to make your life easier. Use a 3-4-5 right triangle. Make the side opposite a 3, the side adjacent to a 4, and the hypotenuse 5. Because sine is $\frac{\text{opposite}}{\text{hypotenuse}}$, $\sin a = \frac{3}{5}$, so $x = \frac{3}{5}$. Cosine is $\frac{\text{adjacent}}{\text{hypotenuse}}$, so $\cos b = \frac{3}{5}$. This is your target; circle it. Make $x = \frac{3}{5}$ in each answer choice and look for the answer that equals $\frac{3}{5}$. Only (A) works, so the correct answer is (A).

4. C

The question asks how tall the flagpole is. Use SOHCAHTOA and your calculator to find the height of the flagpole. From the 70° angle, you know the adjacent side of the triangle, and you want to find the opposite side, so you need to use tangent. $\text{Tangent} = \frac{\text{opposite}}{\text{adjacent}}$, so $\tan 70^\circ = \frac{x}{10\text{m}}$, where x is the height of the flagpole up to the ball. Isolate x by multiplying both sides by 10: $10 \tan 70^\circ = x$. Use your calculator to find that $10 \tan 70^\circ = 27.47$, which is closest to 27. The correct answer is (C).

Summary

◦ Degrees and angles

- A circle contains 360° .
- When you think about angles, remember circles.
- A line is a 180° angle.
- When two lines intersect, four angles are formed; the sum of their measures is 360° .
- When two parallel lines are cut by a third line, the small angles are equal, the big angles are equal, and the sum of a big angle and a small angle is 180° .

◦ Triangles

- Every triangle contains 180° .
- An isosceles triangle is one in which two of the sides are equal in length, and the two angles opposite the equal sides are equal in measure.
- An equilateral triangle is one in which all three sides are equal in length, and all three angles are equal in measure (60°).
- The area of a triangle is $\frac{1}{2}bh$.
- The height of a triangle must form a right angle with the base.
- The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the two legs. Remember the test-writers' favorite Pythagorean triples (3-4-5 and 5-12-13).
- Remember the other special right triangles: 45° - 45° - 90° and 30° - 60° - 90° .
- Similar triangles have the same angles, and their lengths are in proportion.
- Congruent triangles have the same angles and their side lengths are exactly the same.
- For trigonometry questions, remember SOHCAHTOA:

- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

◦ Circles

- The circumference of a circle is $2\pi r$ or πd , where r is the radius of the circle and d is the diameter.
- The area of a circle is πr^2 , where r is the radius of the circle.
- A tangent touches a circle at one point; any radius that touches that tangent forms a 90° angle with the tangent.
- Arcs are proportional to the circumference based on the central angle: $\frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{2\pi r}$.
- Sectors are proportional to the area based on the central angle: $\frac{\text{central angle}}{360^\circ} = \frac{\text{sector area}}{\pi r^2}$.
- If the central angle is given in radians, the measure of the arc is given by $s = r\theta$.

◦ Rectangles and squares

- The perimeter of a rectangle is the sum of the lengths of its sides.
- The area of a rectangle is $\text{length} \times \text{width}$.
- A square is a rectangle whose four sides are all equal in length.
- The volume of a rectangular solid is $\text{length} \times \text{width} \times \text{height}$. The formulas to compute the volumes of other three-dimensional figures are supplied in the reference sheet that can be opened at any time in the testing app.

- When you encounter a geometry question on the Digital SAT, ballpark the answer before trying to work it out.

- Always draw the figure on your scratch paper.
- When information is missing from a diagram, ballpark or plug in.

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Chapter 26

Fill-Ins

On the Digital SAT, approximately 11 of the 44 Math questions will require you to produce your own answer. Although the format of these questions is different from that of the multiple-choice questions, the mathematical concepts tested aren't all that different. In this chapter, we'll show you how to apply what you have learned in the previous chapters to answering fill-in questions.

WHAT IS A FILL-IN?

Both Math modules on the Digital SAT have several questions without answer choices. The exact breakdown per module varies, but you can expect to see about 11 total questions in this format. The test-writers call these Student-Produced Response questions, but we're going to keep things simple and call them fill-ins.

Despite the different format, many of the techniques that you've learned so far still apply to fill-in questions. You can't use Process of Elimination or PITA, of course, but you can still use Plugging In, Bite-Sized Pieces, and other great techniques. Your calculator will still help you out on many of these questions, as well.

The only difficulty with fill-ins is getting used to the way in which you are asked to answer the question. For each fill-in question, you will have a box like this:



To enter your answer, click inside the box and start typing. The numbers you enter will automatically appear left to right, and the testing app will show a preview of your answer so you can make sure it looks right.



Different Format, Same Content

Fill-in questions test the same math topics as multiple-choice questions:

- Algebra
- Advanced Math
- Problem Solving and Data Analysis
- Geometry and Trig

THE INSTRUCTIONS

The fill-in instructions will appear on the left side of the screen for every fill-in question. There are buttons in the middle of the screen that look like this:



You can use the buttons to make the instructions on the left and the question on the right take up more or less of the screen. The instructions and examples in the testing app look like those on the next page.

Student-produced response questions

- If you find **more than one correct answer**, enter only one answer.
- You can enter up to 5 characters for a **positive** answer and up to 6 characters (including the negative sign) for a **negative** answer.
- If your answer is a **fraction** that doesn't fit in the provided space, enter the decimal equivalent.

- If your answer is a **decimal** that doesn't fit in the provided space, enter it by truncating or rounding at the fourth digit.
- If your answer is a **mixed number** (such as $3\frac{1}{2}$), enter it as an improper fraction ($7/2$) or its decimal equivalent (3.5).
- Don't enter **symbols** such as a percent sign, comma, or dollar sign.

Examples

Answer	Acceptable ways to enter answer	Unacceptable: will NOT receive credit
3.5	3.5 3.50 7/2	31/2 3 1/2
$\frac{2}{3}$	2/3 .6666 .6667 0.666 0.667	0.66 .66 0.67 .67
$-\frac{1}{3}$	-1/3 -.3333 -0.333	-.33 -0.33

You don't want to have to spend time rereading the instructions every time, so make sure you know them well. Here is all the information you need to

know about entering a fill-in answer:

1. There is space to enter 5 characters if the answer is positive and 6 characters—including the negative sign—if the answer is negative.
2. You do not need to type the comma for numbers longer than three digits, such as 4,200. In fact, the testing app will not allow it.
3. The testing app also will not allow symbols such as %, \$, or π . Square roots, units, and variables cannot be entered.
4. You can enter your answer as either a fraction or a decimal. For example, .5, 0.5, and $1/2$ are all acceptable answers. Use the forward slash for fractions.
5. If your answer is a fraction, it must fit within the space. Do not try to enter something like $\frac{200}{500}$ as a fraction: either reduce it or convert it to a decimal. Entering $20/50$, $2/5$, .4, and 0.4 would all count as the correct answer.
6. Fractions do not need to be in the lowest reduced form. As long as it fits, it's fine.
7. You cannot fill in mixed numbers. Convert all mixed numbers to improper fractions or decimals. If your answer is $2\frac{1}{2}$, you must convert it to $5/2$ or 2.5. If you enter $21/2$, the testing app will read your answer as $\frac{21}{2}$.

8. If your answer is a decimal that will not fit in the space provided, either enter as many digits as will fit or round the last digit. The fraction $-\frac{2}{3}$ can be entered in decimal form as -0.666 , -0.667 , $-.6666$, or $-.6667$.
9. Some questions will have more than one right answer. Any correct answer you enter will count as correct; do not try to enter multiple answers.

**Fill the Space**

Know what you can and can't enter into the fill-in box:

- 5 characters for a positive answer
- 6 characters for a negative answer
- Don't enter extra zeros if the answer is short
- Do enter as much of a long decimal as will fit
- Don't enter a fraction that doesn't fit
- Do enter reduced fractions that fit

FILL-INS: A TEST DRIVE

To get a feel for this format, let's work through two examples. As you will see, fill-in questions are just regular Digital SAT Math questions.

 Mark for Review

1. If $a + 2 = 6$ and $b + 3 = 21$, what is the value of $\frac{b}{a}$?

Here's How to Crack It

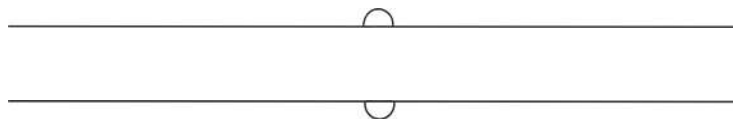
The question asks for the value of $\frac{b}{a}$. You need to solve the first equation for a and the second equation for b . Start with the first equation, and solve for a . By subtracting 2 from both sides of the equation, you should see that $a = 4$.

Now move to the second equation, and solve for b . By subtracting 3 from both sides of the second equation, you should see that $b = 18$.

The question asked you to find the value of $\frac{b}{a}$. That's easy. The value of b is 18, and the value of a is 4. Therefore, the value of $\frac{b}{a}$ is $\frac{18}{4}$.

That's an odd-looking fraction. How in the world do you fill it in? Ask yourself this question: "Does $\frac{18}{4}$ fit?" Yes! Fill in $\frac{18}{4}$.

Your math teacher wouldn't like it, but the scoring computer will. You shouldn't waste time reducing $\frac{18}{4}$ to a prettier fraction or converting it to a decimal. Spend that time on another question instead. The fewer steps you take, the less likely you will be to make a careless mistake.



 Mark for Review

2. The radius of circle O is 212 times the radius of circle P . If the area of circle O is t times the area of circle P , what is the value of t ?

Here's How to Crack It

The question asks for the relationship between the areas of two geometric figures. It doesn't matter that this is a fill-in instead of a multiple-choice question: you still start with the Geometry Basic Approach. Draw two circles on your scratch paper. Next, label the figure. Mark the center of one circle as O and the center of the other as P , and draw in the radius of each circle. The question doesn't give you any numbers for the radius or area of circle P , so plug in. Make the radius 2 and label that on the figure. Finally, write down formulas. The area of a circle is given by $A = \pi r^2$. Plug in $r = 2$ to get $A = \pi(2)^2$, or $A = 4\pi$ for circle P .

The question states that *the radius of circle O is 212 times the radius of circle P* , so multiply 2 by 212 to get $r = (2)(212) = 424$. Label the radius of circle O as 424. Plug $r = 424$ into the area formula to get $A = \pi(424)^2$, or $A = 179,776\pi$. To solve for t , divide the area of circle O by the area of circle P to get $t = \frac{179,776\pi}{4\pi}$, or $t = 44,944$. This is a big number! However, it's still

only 5 characters long, so it will fit in the fill-in box. The fill-in box doesn't accept commas, so don't worry about that. The correct answer is 44944.

44944

MORE POOD

The fill-in questions are mixed in with the multiple-choice questions, and both math modules have an approximate order of difficulty. More important than the question order is your Personal Order of Difficulty (POOD), a strategy that encourages you to focus on the questions you know how to answer first. Don't spend too much time on a question you are unsure about, no matter which format it is.

Keep in mind, of course, that many of the math techniques that you've learned are still very effective on fill-in questions. The Geometry Basic Approach and Plugging In both worked well on the previous question. If you're able to plug in or take an educated guess, go ahead and fill in that answer. As always, there's no penalty for getting it wrong.

Here's another fill-in question that you can answer by using a technique you've learned before.

 Mark for Review

3. Town A has 2,200 residents. The mean age of the residents of Town A is 34. Town B has 3,680 residents with a mean age of 40. What is the mean age of the residents of Town A and Town B combined?

Here's How to Crack It

The question asks for a mean, or average. Work the question in bite-sized pieces and start with Town A. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. The question states that *Town A has 2,200 residents*, so that is the *Number of things*. The question also states that the *mean age of the residents of Town A is 34*, so that is the *Average*. Plug these numbers into the average formula to get $T = (34)(2,200)$, or $T = 74,800$. Do the same thing for Town B: the *Number of things* is 3,680 residents, and the *Average* is the mean age of 40, so the formula becomes $T = (40)(3,680)$, or $T = 147,200$.

Next, add the two totals to get $74,800 + 147,200 = 222,000$. This is the *Total* for the two towns combined. The *Number of things* for the two towns combined is $2,200 + 3,680 = 5,880$ residents. Use the average formula one more time to get $222,000 = (A)(5,880)$. Divide both sides of the equation by 5,880 to get $37.7551020408 = A$.

There clearly isn't room to enter this answer in the fill-in box, so either cut it off or round when you run out of room. You can enter 37.75 or 37.76 and get the question right. Don't round too much, though: if you enter 37.8, you'll get the question wrong. Enter the full five characters to get credit for a positive answer. The correct answer is 37.75 or 37.76.

37.75	or	37.76
------------------------------------	----	------------------------------------

Careless Mistakes

On fill-in questions, you obviously can't use POE to get rid of bad answer choices, and Plugging In the Answers won't work either. In order to earn points on fill-in questions, you're going to have to find the answer yourself, as well as be extremely careful when you enter your answers in the fill-in box. If you need to, double-check your work to make sure you have solved correctly. If you suspect that the question is a difficult one and you get an answer too easily, you may have made a careless mistake or fallen into a trap.

Try the example below with this in mind.



 Mark for Review

4. A teacher is grading two assignments that each had to be a specific length: research papers and short stories. Each research paper has 5 more pages than each short story. How many pages are in a research paper if 7 research papers and 5 short stories have a total of 275 pages?

Here's How to Crack It

The question asks for the number of pages in a research paper given other information about two assignments. Use another skill from earlier in this book and translate English to math in bite-sized pieces. The question states that *each research paper has 5 more pages than each short story*. Let r represent the number of pages in a research paper. The word *has* translates to $=$. The phrase *5 more than* translates to $5 +$. Finally, let s represent the number of pages in a short story. The sentence, therefore, translates to $r = 5$

+ s . Do the same thing with the information that *7 research papers and 5 short stories have a total of 275 pages*. Use r and s again for the number of pages in a research paper and a short story, respectively. Translate *and* as + and *have a total of* as =, and the sentence translates to $7r + 5s = 275$. You now have two equations with the same two variables:

$$r = 5 + s$$

$$7r + 5s = 275$$

Substitute $5 + s$ for r in the second equation to get

$$7(5 + s) + 5s = 275$$

Distribute the 7.

$$35 + 7s + 5s = 275$$

Combine like terms on the left side, then subtract 35 from both sides.

$$12s = 240$$

Isolate s .

$$s = 20$$

It's tempting to fill in 20 and call it a day, but always read the final question! The question asks for the number of pages in a research paper, not in a short story. Plug 20 for s into the first equation to solve for r .

$$r = 5 + 20 = 25$$

Thus, $r = 25$, so fill in that value. The correct answer is 25.

25

MULTIPLE CORRECT ANSWERS

As you've already seen, some fill-in questions will have more than one possible correct answer. It won't matter which correct answer you enter as long as it really is correct. This happens frequently when the answer is a fraction or a decimal. It can also happen when there is more than one solution to an equation.

Let's look at one of those.

 Mark for Review

5. What is one possible solution to the equation $|a + 3| = 7$?

Here's How to Crack It

The question asks for a possible solution to an equation with an absolute value. With an absolute value, the value inside the absolute value bars can be either positive or negative. Set $a + 3$ equal to both 7 and -7 , and solve both equations. When $a + 3 = 7$, subtract 3 from both sides of the equation to get $a = 4$. When $a + 3 = -7$, subtract 3 from both sides of the equation to get $a = -10$. Enter either 4 or -10 and you'll get the question right.

In this case, that was more work than you needed to do. The question asked for *one possible solution*, so you could have stopped after finding one value. However, questions about absolute value might ask for a specific solution—either the positive solution or the negative solution—so always

read the final question (RTFQ) to make sure you don't enter a value that isn't correct.

4	or	-10
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Fill-In Drill

Answers and explanations can be found starting on [this page](#).

 Mark for Review

1. If $a^b = 4$, and $3b = 2$, what is the value of a ?

 Mark for Review

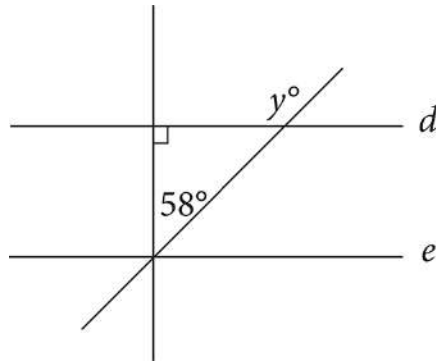
2. If $4x + 2y = 24$ and $\frac{7y}{2x} = 7$, what is the value of x ?

 Mark for Review

$$n = 12(2)^{\frac{t}{3}}$$

3. The number of mice in a certain colony is shown by the formula above, such that n is the number of mice and t is the time, in months, since the start of the colony. If 2 years have passed since the start of the colony, how many mice does the colony contain now?

 Mark for Review



4. In the figure above, if d is parallel to e , what is the value of y ?

 Mark for Review

5. The function g is defined by $g(x) = -(x - 3)(x + 11)$. For what value of x is the value of $g(x)$ at its maximum?

 Mark for Review

6. If line m is defined by the equation $-3x = -2y - 12$ and line n is parallel to line m , what is the slope of line n ?

 Mark for Review

7. If Alexandra pays \$56.65 for a table, and this amount includes a tax of 3% on the price of the table, what is the amount, in dollars, that she pays in tax?

 Mark for Review

8. In triangle ABC , where angle A is a right angle, $\sin(C)$ is $\frac{13}{85}$. What is the value of $\tan(B)$?

 Mark for Review

9. The kinetic energy (KE) of a ball in motion is given by the equation

$KE = \frac{1}{2}mv^2$, where m is the mass of the ball in kilograms (kg) and v is the velocity in meters per second $\left(\frac{\text{m}}{\text{s}}\right)$. A ball with a mass of 5 kg and a kinetic energy of $18.225 \text{ kg} \left(\frac{\text{m}^2}{\text{s}^2}\right)$ is to be rolled along the ground.

What is the velocity of the ball in meters per second, assuming there is no friction?

 Mark for Review

$$x(mx + 42) + 18 = 0$$

10. If the equation above has exactly two real solutions and m is an integer constant, what is the greatest possible value of m ?



FILL-IN DRILL ANSWERS AND EXPLANATIONS

1. 8

The question asks for the value of a . Using $3b = 2$, solve for b by dividing both sides by 3 to get $b = \frac{2}{3}$. That means $a^{\frac{2}{3}} = 4$. Fractional exponents tell you to use the denominator as the root and use the numerator as a regular exponent. So, $\sqrt[3]{a^2} = 4$. First, cube both sides to find $a^2 = 4^3 = 64$. Next, take the square root of both sides to find $a = 8$. The correct answer is 8.

2. 3

The question asks for the value of x in a system of equations. To solve for one variable, find a way to make the other variable disappear when stacking and adding the equations. Start by dividing both sides of the first equation by 2 to get $2x + y = 12$. Multiply both sides of the second equation by 2 to get $7y = 14x$, and then subtract $7y$ from both sides of the equation to get $0 = 14x - 7y$. Now divide both sides of the equation by 7 to get $0 = 2x - y$. The y -terms in the two equations now have the same coefficient with opposite signs, so stack and add the equations.

$$\begin{array}{r} 2x + y = 12 \\ + \quad 2x - y = 0 \\ \hline 4x \quad = 12 \end{array}$$

Divide both sides of the resulting equation by 4 to get $x = 3$. The correct answer is 3.

3. **3072**

The question asks for a value in an equation and gives conflicting units. The question states that t is in months and asks about years.

There are 12 months in 1 year, so there are $(12)(2) = 24$ months in 2 years. Thus, $t = 24$. Plug this value into the given equation to get

$n = 12(2)^{\frac{24}{3}}$, which becomes $n = 12(2)^8$. Use a calculator to get $n =$

$12(256)$, or $n = 3,072$. Leave out the comma when entering the answer in the fill-in box. The correct answer is 3072.

4. **148**

The question asks for the measure of an angle on a figure. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper, including the labels. There is a triangle in the middle of the figure, and all triangles contain 180° . Right angles are 90° , so the third angle measures $180^\circ - 90^\circ - 58^\circ = 32^\circ$. That angle and angle y make up a straight line, and there are 180° in a line, so $32^\circ + y^\circ = 180^\circ$. Subtract 32° from both sides of the equation to get $y^\circ = 148^\circ$. The fill-in box does not allow degree signs, so leave it out. The correct answer is 148.

5. **-4**

The question asks for the value when a quadratic function reaches its maximum. A parabola reaches its minimum or maximum value at its

vertex, so find the x -coordinate of the vertex. One method is to enter the equation into a graphing calculator, then scroll and zoom as needed to find the vertex. The vertex is at $(-4, 49)$, so the value of the x -coordinate is -4 . Another way to solve is to recognize that the vertex of a parabola is on the axis of symmetry, which is the midpoint between the two x -intercepts. Find the x -intercepts by setting each factor equal to 0 and solving. When $x - 3 = 0$, $x = 3$. When $x + 11 = 0$, $x = -11$. The x -coordinate of the midpoint is the average of the two x -intercepts, which is $\frac{3 + (-11)}{2} = \frac{-8}{2} = -4$. Using either method, the correct answer is -4 .

6. $\frac{3}{2}$ or 1.5

The question asks for the slope of a line. The question states that line m and line n are parallel, which means they have the same slope but different y -intercepts. The question gives the equation of line m , so find the slope of that line. Rearrange the equation so it is in slope-intercept form, $y = mx + b$, in which m is the slope and b is the y -intercept. Add $2y$ to both sides of the equation to get $-3x + 2y = -12$, then add $3x$ to both sides of the equation to get $2y = 3x - 12$. Finally, divide both sides of the equation by 2 to get $y = \frac{3}{2}x - 6$. The slope of line m is $\frac{3}{2}$, so the slope of line n is also $\frac{3}{2}$. The answer can be entered

as a fraction or in decimal form, which is 1.5. The correct answer is $\frac{3}{2}$ or 1.5.

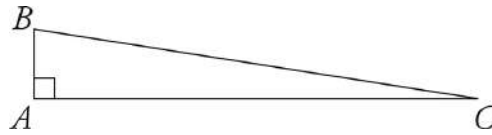
7. **1.65**

The question asks for a value given a specific situation. Translate the information in bite-sized pieces. The question states that *Alexandra pays \$56.65 for a table*, which includes *a tax of 3% on the price of the table*. The unknown is the price of the table before tax, so make that x . *Percent* means out of 100, so translate 3% as $\frac{3}{100}$. The tax is added to the cost before tax, so add those two values and set them equal to the total cost of \$56.65. The equation becomes $x + \left(\frac{3}{100}\right)x = \56.65 . Simplify the left side of the equation to get $x + 0.03x = \$56.65$, and then $1.03x = \$56.65$. Divide both sides of the equation by 1.03 to get $x = \$55.00$. This is the cost before tax. The question asks for the amount of tax, *in dollars*, so subtract the cost before tax from the final cost to get $\$56.65 - \$55.00 = \$1.65$. The fill-in box does not allow dollar signs, so leave it out. The correct answer is 1.65.

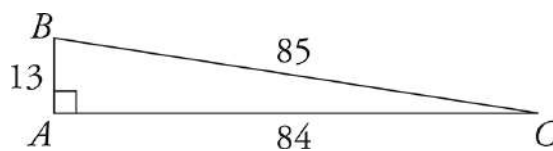
8. $\frac{84}{13}$ or **6.461** or **6.462**

The question asks for the value of a trigonometric function. Use the Geometry Basic Approach. Begin by drawing a right triangle. Next,

label the vertices and label the right angle as angle A. The drawing should look something like this:



Next, write out SOHCAHTOA to remember the trig functions. The SOH part defines the sine as $\frac{\text{opposite}}{\text{hypotenuse}}$, and the question states that $\sin(C) = \frac{13}{85}$. Label the side opposite angle C, which is $\overline{AB}$, as 13 and the hypotenuse, which is $\overline{BC}$, as 85. To find the length of the third side, use the Pythagorean Theorem: $a^2 + b^2 = c^2$. Plug in the known values to get $13^2 + b^2 = 85^2$. Square the numbers to get $169 + b^2 = 7,225$, then subtract 169 from both sides of the equation to get $b^2 = 7,056$. Take the square root of both sides of the equation to get $b = 84$. With all three side lengths labeled, the drawing looks like this:



To find $\tan(B)$, use SOHCAHTOA again. The TOA part defines the tangent as $\frac{\text{opposite}}{\text{adjacent}}$. The side opposite angle B is 84, and the side adjacent to angle B is 13, so $\tan(B) = \frac{84}{13}$. The answer can also be entered in the fill-in box as a decimal. In this case, $\frac{84}{13} = 6.\overline{461538}$, which is too long. Either stop when there's no more room and enter

6.461 or round the last digit and enter 6.462. The correct answer is $\frac{84}{13}$ or equivalent forms.

9. 2.7

The question asks for a value given an equation. Plug in the values given in the question and solve for the other value. The question states that the ball has a mass of 5 kg, and that m represents mass, so plug in $m = 5$. The question also states that the kinetic energy is 18.225, and that KE represents kinetic energy, so plug in $KE = 18.225$. The equation becomes $18.225 = \frac{1}{2}(5)(v)^2$. Multiply both sides of the equation by 2 to get $36.45 = (5)(v)^2$. Divide both sides of the equation by 5 to get $7.29 = v^2$, then take the square root of both sides of the equation to get $2.7 = v$. The correct answer is 2.7.

10. 24

The question asks for the value of a constant in a quadratic equation. Start by distributing the x on the left side of the equation to get $mx^2 + 42x + 18 = 0$. The question states that the system *has exactly two real solutions*. To determine the number of solutions to a quadratic, use the discriminant. The discriminant is the part of the quadratic formula under the square root sign, and it can be written as $D = b^2 - 4ac$. When the discriminant is positive, the quadratic has exactly two real solutions; when the discriminant is 0, the quadratic has exactly one real solution; and when the discriminant is negative, the quadratic has

no real solutions. Since this quadratic has exactly two real solutions, the discriminant must be positive.

The quadratic is now in standard form, $ax^2 + bx + c = 0$, so $a = m$, $b = 42$, and $c = 18$. Plug these into the discriminant formula to get $D = 42^2 - 4(m)(18)$, which becomes $D = 1,764 - 72m$. In order for D to be positive, the result must be greater than 0, so write the inequality $1,764 - 72m > 0$. Add $72m$ to both sides of the inequality to get $1,764 > 72m$, and then divide both sides of the inequality by 72 to get $24.5 > m$. The question states that m is an integer, so the greatest possible value of m is 24. The correct answer is 24.

Summary

- Both of the Math modules on the Digital SAT contain several questions without answer choices. The test-writers call these questions “student-produced responses.” We call them fill-ins because you have to fill in your own answer.
- Despite their format, fill-ins are really just like other Math questions on the Digital SAT, and many of the same techniques that you have learned still apply.
- The fill-in questions and multiple-choice questions are mixed together in a loose order of difficulty. Use your knowledge of your own strengths and weaknesses to decide which ones to tackle first and which ones, if any, to skip.
- The fill-in format increases the likelihood of careless errors. Know the instructions and check your work carefully.
- Just like the rest of the Digital SAT, there is no guessing penalty for fill-ins, so you should always fill in an answer, even if it’s a guess.
- Enter only one answer even if the question has multiple possible answers. It doesn’t matter which answer you enter, as long as it’s one of the possible answers.
- Enter up to 5 characters when the answer is positive. Enter up to 6 characters, including the negative sign, when the answer is negative.
- The characters that can be entered are the digits 0–9, the negative sign, the forward slash (/) for fractions, and the decimal point. Special characters such as % or π cannot be entered.

- If the answer to a fill-in question contains a fraction or decimal, you can enter the answer in either form. Use whichever form is easier and less likely to cause mistakes.
- If your answer is a fraction that doesn't fit in the space, either reduce the fraction or convert it to a decimal.
- If a fraction fits in the space, you don't have to reduce the fraction before entering it.
- Do not enter mixed numbers. Convert mixed numbers to fractions or decimals before entering your answer.
- If your answer is a long or repeating decimal, fill up all of the space. Either keep entering digits until the space is full or round the last digit that will fit.

Part IV

Taking the Digital SAT

THE DIGITAL SAT IS A WEEK AWAY—WHAT SHOULD YOU DO?

First of all, you should practice the techniques we’ve taught you on any practice test you take. If you haven’t done so already, take and score one or more of the practice tests in this book and online. You can also download a practice test from College Board’s website, satsuite.collegeboard.org/digital/digital-practice-preparation.

If you want more practice, pick up a copy of our very own *645+ Practice Questions for the Digital SAT* at your local bookstore or through our website at PrincetonReview.com/bookstore.



Perfect Your Skills

In addition to taking the practice tests in this book, you should register your book (See “Get More (Free) Content” on [this page](#)) to gain access to your Student Tools for even more practice, as well as other fantastic resources to enhance your prep.

Getting Psyched

The Digital SAT is a big deal, but don’t let it scare you. Sometimes students get so nervous about doing well that they worry they will freeze up on the test and ruin their scores. The best thing to do is to think of the Digital SAT as a game. It’s a game you can get better at, and beating the test can be fun. When you go into the test center, just think about the advantage you have because you know how to plug in when you see variables in the answer choices.

The best way to keep from getting nervous is to build confidence in yourself and in your ability to remember and use our techniques. When you take practice tests, time yourself exactly as you will be timed on the real Digital SAT. Develop a sense of how long 35 minutes is, for example, and how much time you can afford to spend on cracking difficult questions. If you know ahead of time what to expect, you won't be as nervous.

Of course, taking a real Digital SAT is much more nerve-racking than taking a practice test. Prepare yourself ahead of time for the fact that 35 minutes will seem to go by a lot faster on a real Digital SAT than it did on your practice tests.

It's all right to be nervous; the point of being prepared is to keep from panicking.

Should You Sleep for 36 Hours?

Some guidance counselors tell their students to get a lot of sleep the night before the Digital SAT. This probably isn't a good idea. If you aren't used to sleeping 12 hours a night, doing so will just make you groggy for the test. The same goes for going out all night: tired people are not good test-takers.

A much better idea is to get up early each morning for the entire week before the test and do your homework before school. This will get your brain accustomed to functioning at that hour of the morning. You want to be sharp at test time.

Before dinner the night before the test, spend an hour or so reviewing or doing a few practice problems. The goal here is to brush up on the material, not to exhaust yourself by over-cramming.



Veg Out

Although preparation is key to doing well on the test, you shouldn't exhaust yourself trying to cram information into your head. Take some breaks between study sessions to relax, unwind, and rest your mind.

Furthermore...

Here are a few pointers for test day and beyond:

1. Eat a good breakfast before the test—your brain needs energy.
2. Work out a few Digital SAT questions on the morning of the test to help dust off any cobwebs in your head and get you to start thinking analytically.
3. Arrive at the test center early. Everyone is headed to the same place at the same time.
4. You must take acceptable identification to the test center on the day of the test. Acceptable identification must include a recognizable photograph and your name. Acceptable forms of ID include your driver's license, a school ID with a photo, or a valid passport. If you don't have an official piece of ID with your signature and your photo, you can have your school make an ID for you using a Student ID form provided by College Board. Complete instructions for making such an ID are found on College Board's website. According to College Board, the following forms of ID are *unacceptable*: a birth certificate, a credit card, or a Social Security card. Make sure you read all of the rules in the Student Registration Booklet because conflicts with the College

Board are just not worth the headache. Your only concern on the day of the test should be doing well on the Digital SAT. To avoid hassles and unnecessary stress, make *absolutely certain* that you take your admissions ticket and your ID with you on the day of the test.

1. Eat Breakfast

You'll work better on a satisfied stomach.

2. Try Some Questions

Get your mind moving.

3. Show Up Early

Leave time for traffic.

4. Take Your ID

A driver's license, a passport, or a school photo ID will do.

5. If you are taking the Digital SAT on your own laptop or tablet, you will need to bring it to the testing center. Make sure it is fully charged, and bring a charging cable—but there's no guarantee you'll have access to an outlet. If you are using a device from the testing center or from College Board, leave your smart devices at home. Cell phones are permitted as long as they are put away and turned off, but we recommend leaving them at home or in a car, if at all possible, because if your phone somehow makes a noise or you touch or look at it by accident, you could have your test canceled.

5. No Extra Technology

Only bring a laptop or tablet if you need it for testing. Don't wear a smartwatch, and leave your phone at home or in the car if possible!

6. The only outside tool you are allowed to use on the test is a calculator. The Digital SAT has a built-in calculator for all Math questions, but you may be more comfortable with your own calculator. If so, make sure it is on the approved calculator list and has fresh batteries before the test.

6. Remember Supplies

Bring your calculator if you plan to use it, and bring extra batteries just in case.

7. Some proctors allow students to bring food into the test room; others don't. Take a snack like a banana, which is sure to give you an energy boost. Save it until your break and eat it outside the test room.

7. Take Fruit or Other Energy Food

Grapes or oranges can give you an energy boost if you need it.

8. You are going to be sitting in the same place for a little over two hours, so make sure your desk isn't broken or unusually uncomfortable. If you are left-handed, ask for a left-handed desk. (The center may not have one, but it won't hurt to ask.) If the Sun is in your eyes, ask to move. If the room is too dark, ask someone to turn on the lights. Don't hesitate to speak up.

8. Your Desk...

should be comfortable and suited to your needs.

9. Make sure to follow all directions from College Board about how to be ready to go at testing time. You will have to ensure that your device has the testing application and that you can access the test. Tech help will be available if you need it.

9. Your Testing App...

should be functioning properly, but ask for tech help if needed.

10. Make sure to click on an answer for every question. Use the module Review tool at any time during the module and again at the end of the

module to see which questions remain marked, unanswered, or both.

10. Click Carefully

Make sure to select the correct answer to each question after doing your work. Enter answers to student-produced response questions and verify correct entry with the answer preview.

11. You deserve to take your Digital SAT under good conditions. While you are testing, if there is an issue that can be addressed immediately (the high school band is practicing outside the window or your chair is wobbly), see if you can get the proctor to fix the situation. If not, you can report the conditions to College Board, but do all you can to ensure good testing conditions while at the testing center.

11. We're Here for You

The Princeton Review is proud to advise students who feel their exam was not administered properly.

This test is challenging, but you've put in the time and effort to succeed.

You've got this!

OceanofPDF.com

Part V

Practice Test 2

[Practice Test 2](#)

[Practice Test 2: Diagnostic Answer Key and Explanations](#)

[*OceanofPDF.com*](#)

Practice Test 2

[Click here](#) to download a PDF of Practice Test 2 with bubble sheet.

The Digital SAT will be administered on a computer or tablet, so it is best if you take your practice tests in the online Student Tools for this book. However, if you are unable to test on a computer or if you have accommodations and will take the official test on paper, you may take Test 2 on paper in this book instead. Both sets of instructions are below.

To Test Online:

Register your book according to the instructions on [this page](#). In your Student Tools, you will be able to access the tests associated with this book: both the two printed in this book and the additional online-only tests. Taking these online adaptive tests is a great way to prepare for taking the actual Digital SAT.

The Digital SAT has only two modules in each section, not three like the test printed in this book. The second module you get in each section will be determined by your performance on the first module in that section. The online tests follow this structure, and once you finish the test, you will get an estimated score based on the modules you saw and the questions you got right.

To Test on Paper:

For both RW and Math, the following test contains a standard first module and two options for the second module, one easier and one harder. You should take the appropriate second module based on your performance in the first module, as detailed below, but you can feel free to use the other module for extra practice later.

In order to navigate the practice test in this book, take the following steps. To record your answers, you can either indicate them as described in the directions for print tests included with each module or by entering them onto the answer sheet on [this page](#).

- ☐ Take Reading and Writing (RW) Module 1, allowing yourself 32 minutes to complete it.
- ☐ Go to the answer key starting on [this page](#) and determine the number of questions you got correct in RW Module 1.
- ☐ If you get fewer than 15 questions correct, take RW Module 2 – Easier, which starts on [this page](#). If you get 15 or more questions correct, take RW Module 2 – Harder, which starts on [this page](#).
- ☐ Whichever RW Module 2 you take, start it immediately and allow yourself 32 minutes to complete it.
- ☐ Take a 10-minute break between RW Module 2 and Math Module 1.
- ☐ Take Math Module 1, allowing yourself 35 minutes to complete it.
- ☐ Go to the answer key starting on [this page](#) and determine the number of questions you got correct in Math Module 1.
- ☐ If you get fewer than 14 questions correct, take Math Module 2 – Easier, which starts on [this page](#). If you get 14 or more questions correct, take Math Module 2 – Harder, which starts on [this page](#).
- ☐ Whichever Math Module you take, start it immediately and allow yourself 35 minutes to complete it.
- ☐ After you finish the test, check your answers to RW Module 2 and Math Module 2.
- ☐ Only after you complete the entire test should you read the explanations for the questions, which start on [this page](#) and are also available online.
- ☐ Go to your online Student Tools to see the latest information about scoring and to get your estimated score.

Section 1, Module 1: Reading and Writing

SAT Prep Test 2—Reading and Writing Module 1

Turn to Section 1 of your answer sheet ([this page](#)) to answer the questions in this section.

DIRECTIONS

The questions in this section address a number of important reading and writing skills. Each question includes one or more passages, which may include a table or graph. Read each passage and question carefully, and then choose the best answer to the question based on the passage(s).

All questions in the section are multiple-choice with four answer choices. Each question has a single best answer.

 Mark for Review

1. The Chilean volcano Calabozos is located in _____ area.
Therefore, the risk of loss of human life in the event of an eruption is minimal.

Which choice completes the text with the most logical and precise word or phrase?

☐ A a hazardous

☐ B an active

☐ C a mountainous

☐ D a remote

 Mark for Review

2. Contemporaries of American modernist poet H.D. focused only on her important contributions to the Imagist movement in the 1920s, taking _____ view of her work. However, she wrote in a variety of forms and genres, from short, lyrical works to complex, book-length poems.

Which choice completes the text with the most logical and precise word or phrase?

☐ A an expansive

☐ B a limited

☐ C an imaginative

☐ D a complicated

 Mark for Review

3. Since the 1950s, scientists have known that rapid eye movement, or REM, occurs when someone is sleeping. Previous studies attempting to determine the meaning of these eye movements have been unsuccessful in part because these studies relied on human subjects recalling the content of their dreams. A recent study by physiologists Yuta Senzai and Massimo Scanziani has avoided this issue by studying dreaming mice instead. Their results suggest that REM is correlated to changes in direction during the dream.

Which choice best describes the function of the second sentence in the overall structure of the text?

☐ A It names a problem in the approach taken by Senzai and Scanziani.

☐ B It introduces the difficulty that the study by Senzai and Scanziani was designed to bypass.

☐ C It presents the findings of studies done prior to the study by Senzai and Scanziani.

☐ D It clarifies how others studying REM sleep interpret the study by Senzai and Scanziani.

 Mark for Review

4. Electoreception is the ability of an animal to sense the flow of electricity around it by using specialized organs known as electoreceptors. Most species known to use electoreception are fish, including many sharks, elephant fishes, and eels. However, electoreception is not limited to fish. Monotremes, a group of mammals that includes the platypus and some echidnas, have electoreceptors on or near their mouths to help locate prey. There is also some evidence that bees can detect static electricity on flowers.

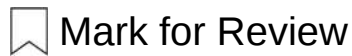
Which choice best describes the function of the third sentence in the overall structure of the text?

☐ A It generalizes the phenomenon discussed beyond fishes.

☐ B It offers another explanation of electoreception that is different from the explanation of how electoreception is used by fishes.

☐ C It provides more examples of animals with electoreception.

☐ D It explains how electoreception evolved in monotremes and bees.



Text 1

An animal is said to have a theory of mind when it is able to act according to the mental states of other individuals. Psychologists David Premack and Guy Woodruff studied whether chimpanzees have such a theory of mind. They showed videos of human actors struggling with various problems. The chimpanzees were able to select photographs that showed the best tool to solve each actor's problem.

Text 2

Biologist Daniel J. Povinelli and psychologists Kurt E. Nelson and Sarah T. Boysen have argued that previous research into whether chimpanzees have a theory of mind have not adequately addressed alternative explanations for the chimpanzees' behaviors. Specifically, it may be the case that chimpanzees are following learned behaviors in a known environment, rather than applying a theory of mind in a novel situation.

5. Based on the texts, how would Povinelli, Nelson, and Boysen (Text 2) most likely respond to Premack and Woodruff (Text 1)?

☐ A They would argue that nonhuman primates other than chimpanzees, such as baboons and gorillas, may also have a theory of mind.

☐ B They would argue that the chimpanzees would be able to solve the problems themselves without referencing the photographs by struggling with the situation themselves and eventually determining the correct solution.

☐ C They would encourage Premack and Woodruff to show the same videos and photographs to other nonhuman primates

and compare the other nonhuman primates' reactions to the chimpanzees' reactions.

- ☐ D They would suggest that placing the chimpanzee subjects in novel environments, such as rooms distinct from the chimpanzees' regular enclosures, may help better ascertain whether chimpanzees have a theory of mind.

 Mark for Review

6. The following text is from Oscar Wilde's 1890 novel *The Picture of Dorian Gray*. Dorian is seeing his portrait, painted by Basil Hallward, for the first time.

Dorian made no answer, but passed listlessly in front of his picture and turned towards it. When he saw it he drew back, and his cheeks flushed for a moment with pleasure. A look of joy came into his eyes, as if he had recognized himself for the first time. He stood there motionless and in wonder, dimly conscious that Hallward was speaking to him, but not catching the meaning of his words. The sense of his own beauty came on him like a revelation. He had never felt it before.

According to the text, what is true about Dorian?

☐ A Dorian is distracted by the beauty of the painting.

☐ B Dorian believes that what Hallward is saying is unimportant.

☐ C Dorian does not recognize his own image.

☐ D Dorian is prone to embarrassment.

 Mark for Review

7. The following text is from Frederick Marryat's 1847 novel *The Children of the New Forest*.

The old forester lay awake the whole of this night, reflecting how he should act relative to the children; he felt the great responsibility that he had incurred, and was alarmed when he considered what might be the consequences if his days were shortened. What would become of them—living in so sequestered a spot that few knew even of its existence—totally shut out from the world, and left to their own resources?

Based on the text, what is true about the children?

- ☐ A They are isolated from people other than the old forester.
- ☐ B They are completely unable to take care of themselves.
- ☐ C The old forester is resentful of having to take care of them.
- ☐ D They attempt to help the old forester with his responsibilities.

 Mark for Review

8. The following text is Baron George Gordon Byron's poem "Answer to _____'s Professions of Affection," written around 1814. The poem is addressed to an unknown person.

In hearts like thine ne'er may I hold a place Till I renounce all sense, all shame, all grace—That seat,—like seats, the bane of Freedom's realm, But dear to those presiding at the helm—Is basely purchased, not with gold alone; Add Conscience, too,

this bargain is your own—‘Tis thine to offer with corrupting art
The rotten borough of the human heart.

What is the main idea of the text?

☐ A The speaker is expressing disapproval towards the unknown person.

☐ B The speaker is unimportant to the unknown person.

☐ C The speaker is thinking of purchasing a seat.

☐ D The speaker holds a place in the heart of the unknown person.

 Mark for Review

9. Sepsis is a life-threatening condition caused by the body’s response to an infection. These infections are typically bacterial but may be fungal, parasitic, or viral. The body’s response to these infections leads to increased inflammation and organ damage. This damage, in turn, results in a weakened immune system, which increases the likelihood of reinfection. In a recent study, a team of doctors and pharmacologists led by Shubham Soni claims that administering ketone esters can reduce inflammation and immune system weakening caused by sepsis.

Which finding from the team led by Soni, if true, would most directly support its claim?

☐ A Patients with sepsis who were administered ketone esters had fewer signs of inflammation and less organ damage than those administered standard antibiotics.

☐ B When administered, ketone esters are known to increase blood ketone levels, which in turn are a source of energy for the brain.

☐ C Both those patients administered ketone esters and those administered standard antibiotics did not have reduced inflammation when treated with medication intended to reduce fever.

☐ D Those sepsis patients administered ketone esters had reduced inflammation but greater organ damage than those administered standard antibiotics.

 Mark for Review

Horses' Responses to Novel Objects Based on Number of Handlers

	Only One Handler	Multiple Handlers
No reluctance	45%	25%
Mild reluctance	42%	49%
Strong reluctance	13%	26%

10. Horses have been domesticated for thousands of years. Therefore, they show great sensitivity to the emotions of humans. Biologist Océane Liehrmann from the University of Turku, Finland, led a team of researchers in a study of horses to determine the effect of the number of handlers (either only one person or multiple people) on the horses' responses to a novel object. The researchers determined that horses with only one handler were less reluctant to interact with the novel object than were horses with multiple handlers. For example,

45% of horses with only one handler had no reluctance when interacting with a novel object while _____.

Which choice most effectively uses data from the table to complete the example?

☐ A 13% of horses with only one handler had strong reluctance.

☐ B 25% of horses with multiple handlers had no reluctance.

☐ C 26% of horses with multiple handlers had strong reluctance.

☐ D 42% of horses with only one handler had mild reluctance.

 Mark for Review

Indian Lok Sabha Results by Percentage of Seats Won, 1999–2019

Party	1999	2004	2009	2014	2019
Bharatiya Janata Party	33%	25%	21%	52%	56%
Indian National Congress	21%	27%	38%	8%	10%
Communist Party of India (Marxist)	6%	8%	4%	7%	4%
Other	40%	40%	37%	33%	30%

11. India is the largest democracy in the world, with over 614 million people voting in the 2019 election for the Lok Sabha, the parliament of the federal government. In the early years of Indian independence, from the first election in 1951–52 through the eighth Lok Sabha in 1984, each election resulted in one party winning the majority of seats. However, starting with the 1989 election, the party with the largest number of seats failed to win more than half of the total seats. This trend was eventually broken by the Bharatiya Janata Party, which _____.

Which choice most effectively uses data from the graph to illustrate the claim?

☐ A went from holding the second most seats among the top 3 parties in parliament in 2004 and 2009 to holding a majority of seats in 2014 and 2019.

☐ B reached its highest percentage of seats the same year that the Indian National Congress had its lowest percentage of seats over the same time period.

☐ C won a lower percentage of seats in the 2009 election than in the 2004 election.

☐ D had a lower percentage of seats than the Indian National Congress in 2004 but a higher percentage of seats than the Indian National Congress in 1999.

 Mark for Review

Changes in Indicators of Fatty Liver Disease in Vitamin B12 and Placebo Groups

Indicator	Vitamin B12 Group	Control Group
steatosis values (dB/cm/MHz)	−0.41	−0.30
fibrosis values (kPa)	−0.35	0.10
fasting blood glucose (mg/dl)	−5.00	−1.50
fasting serum insulin (μU/ml)	−1.46	−0.21
homeostasis model assessment of insulin resistance (HOMA-IR)	−0.23	0.06

12. Fatty liver disease (FLD) occurs when excess fat builds up in the liver. While there are often few or no symptoms of FLD, if left untreated, it can lead to cirrhosis or liver cancer. Because FLD is often asymptomatic, doctors and researchers rely on indicators such as steatosis (retention of fat in the liver), fibrosis (scarring), blood glucose (sugar), serum insulin, and insulin resistance to measure and track the development of FLD. A group of researchers led by radiologist Hamid Reza Talari hypothesized that those who take vitamin B12 would experience improvements in fibrosis and insulin resistance when compared to a control group over the same time period.

Which choice best describes data from the table that support the researchers' hypothesis?

☐ A Those in the control group had decreases in their steatosis values and fasting blood glucose but had increases in fibrosis values and HOMA-IR.

☐ B Those in the vitamin B12 group had decreases in fibrosis values and HOMA-IR levels, whereas those in the control group had increases in these same values.

☐ C Both those in the vitamin B12 group and the control group had decreases in their steatosis values.

☐ D Those in the control group had a decrease in their fasting blood glucose, but those in the vitamin B12 group had an increase in their fasting blood glucose.

 Mark for Review

Mean Levels of Carbon Monoxide (ppm), November 18–26, 1966

City	Day of the Month of November								
	18	19	20	21	22	23	24	25	26
Newark, NJ	16	14	15	21	23	28	32	27	21
New York, NY	4	3	1	2	3	6	7	13	8
Philadelphia, PA	6	0	0	0	1	6	9	10	6
Washington, D.C.	4	2	3	0	0	0	0	0	0

13. The air pollution produced in an area is only one factor in that area's air quality. Weather patterns, in particular wind and the movement of air masses, can affect the concentration of pollutants such as carbon monoxide. During a smog event that occurred in the northeastern United States in November 1966, levels of carbon monoxide were recorded in Newark, New Jersey, the origin of the smog event, as well as neighboring city New York, NY, and more distant cities such as Philadelphia, PA, and Washington, D.C. The localized nature of weather patterns during this event can be seen by comparing Newark, NJ, and New York, NY, with _____.

Which choice most effectively uses data from the table to complete the statement?

☐ A Washington, D.C., on the 18th and the 19th.

☐ B Philadelphia, PA, on the 23rd and the 25th

☐ C Philadelphia, PA, on the 24th and the 26th.

☐ D Washington, D.C., on the 23rd and the 24th.

 Mark for Review

14. Neurons respond to stimuli from sensory organs or other neurons. Learning occurs when neurons change how they respond to stimuli based on previous experience, which is a property of memory. Electrical engineers seek to replicate similar processes in their development of computer memory. Recently, research by electrical engineer Mohammad Samizadeh Nikoo has demonstrated that vanadium dioxide (VO₂) has a similar memory property to that of neurons, suggesting that _____.

Which choice most logically completes the text?

☐ A VO₂ could be used in the development of computer memory.

☐ B neurons use VO₂ when forming memories.

☐ C VO₂ can learn to respond to stimuli from sensory organs.

☐ D electrical engineers can now use neurons to develop computer memory.

 Mark for Review

15. Uruguayan-Spanish author Carmen Posadas has written the children's books *Juego de Niños* (*Child's Play*) and *La Cinta Roja* (*The Red Ribbon*). Currently, _____ available in over fifty countries and thirty languages.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A some are

☐ B this is

☐ they are

☐ it is

 Mark for Review

16. During a meeting, a group of twelve young deaf people shared their feelings of isolation and their desire for support. In 1988, the group worked together to form Action Deaf Youth, an _____ provides services and programs for deaf children and youth throughout Northern Ireland.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ organization, that

☐ organization

☐ organization that

☐ organization,

 Mark for Review

17. In 1986, after a 56-day expedition, Ann Bancroft became the first woman to reach the North Pole. Her experience as a physical education teacher and her leadership of the first all-female team to cross the ice to the South _____ her to create a foundation that supports girls in pursuing their dreams.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A Pole to inspire

☐ B Pole that inspired

☐ C Pole, inspiring

☐ D Pole inspired

 Mark for Review

18. American artist Simone Leigh creates art in various mediums, including sculptures, video, and _____ the themes and images in her artwork, Leigh has emphasized that Black women are her primary audience and that they would be familiar with the allusions in her work.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A performance. Discussing

☐ B performance discussing

☐ C performance and discussing

☐ D performance, discussing

 Mark for Review

19. Japanese origamist Akira Yoshizawa is considered the grandmaster of origami, creating more than 50,000 models as well as wet-folding, the most well-known of his invented techniques. _____ dampening the paper before folding, leading to origami models with rounder and more sculpted looks.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A It involves

☐ B They involve

☐ C One involves

☐ D These involve

 Mark for Review

20. Chinese artist Xu Bing is known for his art installations that showcase his printmaking skills and his creative use of languages and texts. His 1991 installation *A Book from the Sky*, for example, consists of volumes and scrolls printed with characters he invented, while his 2004 installation *The Glassy Surface of a _____* uses the text of Henry David Thoreau's *Walden* to create the illusion of a lake.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A *Lake:*

☐ B *Lake*

☐ C *Lake,*

☐ D *Lake—*

 Mark for Review

21. Developed along with the swing style of jazz music in the 1920s, swing dance is a group of social dances that once comprised hundreds

of styles. Not all of the styles survived beyond that time _____ the dances that are still popular today include Lindy Hop, Balboa, Collegiate Shag, and Charleston.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A period; however,

☐ B period, however;

☐ C period, however,

☐ D period, however



Mark for Review

22. Evolutionary biologist Jonathan Cade may have discovered the oldest amphibious beaver species in the world. Cade first compared measurements of the beaver's ankle to those of almost 350 other rodent species to learn more about how it moved. _____ Cade dated the species to approximately 30 million years ago based on its location between rock and ash layers.

Which choice completes the text with the most logical transition?

☐ A For example,

☐ B In conclusion,

☐ C Next,

☐ D In fact,



Mark for Review

23. Male and female American citizens had starkly different roles during World War II. Men served as soldiers or took part in the workforce to create weapons and other wartime materials. _____ women were responsible for maintaining the home and supporting the men. Some women also ventured into the workforce for the first time, and the famous “We Can Do It” poster featuring “Rosie the Riveter” was created to motivate women to pursue this new role.

Which choice completes the text with the most logical transition?

☐ A Besides,

☐ B Instead,

☐ C Likewise,

☐ D Meanwhile,

 Mark for Review

24. While treatment for hearing loss is typically associated with the ears, some patients with damaged ear structures are not able to use traditional cochlear implants. _____ researchers are working to develop hearing aids anchored to patients’ bones in order to combat hearing loss through vibrations in the skull.

Which choice completes the text with the most logical transition?

☐ A Secondly,

☐ B In addition,

☐ C Finally,

☐ D Hence,

 Mark for Review

25. Korean artist Anicka Yi uses a unique process and materials to generate her art installations. Her materials are often perishable and biological, such as soap and flowers, and are not traditionally used for artwork. _____ Yi spends almost as much time transforming these substances into completely new materials as she does creating the actual art pieces.

Which choice completes the text with the most logical transition?

☐ A Meanwhile,

☐ B Instead,

☐ C In fact,

☐ D To conclude,

 Mark for Review

26. While researching a topic, a student has taken the following notes:

- A writing system for expressing numbers is a numeral system.
- Two examples of numeral systems from history are Babylonian cuneiform numerals and Roman numerals.
- The Babylonian cuneiform numeral system is a base-60 system and lacks a zero digit.
- It's a positional numeral system in which the position of a digit affects its value.
- The Roman numeral system is a base-10 system and lacks a zero digit.

- It's a non-positional numeral system in which the position of a digit does not affect its value.

The student wants to emphasize a difference between the two numeral systems. Which choice most effectively uses relevant information from the notes to accomplish this goal?

☐ A Babylonian cuneiform numerals and Roman numerals are two writing systems for expressing numbers.

☐ B The Roman numeral system is a base-10 non-positional system that lacks a zero digit.

☐ C One system for expressing numbers is Babylonian cuneiform; however, another one is the Roman numeral system.

☐ D The Babylonian cuneiform numeral system is base-60 and positional, while the Roman numeral system is base-10 and non-positional.

 Mark for Review

27. While researching a topic, a student has taken the following notes:

- Archaeologists studied the burial of an individual at the Newen Antug site in Argentinian Patagonia.
- The individual was buried in a wooden structure over 800 years ago.
- An analysis of the structure revealed that it was carved from a tree with excellent buoyancy.
- The wooden structure was a canoe, suggesting that canoes were used as coffins at that time.

The student wants to present the Newen Antug study and its conclusions. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- ☐ A The burial site of an individual over 800 years ago was found at the Newen Antug site in Argentinian Patagonia.
- ☐ B Archaeologists studied the burial site of an individual who was buried at the Newen Antug site over 800 years ago.
- ☐ C An analysis of a burial site at the Newen Antug site in Argentinian Patagonia provided evidence that canoes were used as coffins over 800 years ago.
- ☐ D As part of a study of a burial site at the Newen Antug site in Argentinian Patagonia, a wooden structure buried with an individual was analyzed.

YIELD

Once you've finished (or run out of time for) this section, use the answer key to determine how many questions you got right. If you got fewer than 15 questions right, move on to Module 2—Easier, otherwise move on to Module 2—Harder.

Section 1, Module 2—Easier: Reading and Writing

SAT Prep Test 2—Reading and Writing

Module 2—Easier

Turn to Section 1 of your answer sheet ([this page](#)) to answer the questions in this section.

DIRECTIONS

The questions in this section address a number of important reading and writing skills. Each question includes one or more passages, which may include a table or graph. Read each passage and question carefully, and then choose the best answer to the question based on the passage(s).

All questions in the section are multiple-choice with four answer choices. Each question has a single best answer.

 Mark for Review

1. Shakespeare intentionally provided no stage directions for his play *Macbeth* regarding whether to have Banquo’s ghost physically present on stage or simply to have Macbeth react fearfully to something invisible, thus providing future directors with the _____ to indulge their own artistic interpretations.

Which choice completes the text with the most logical and precise word or phrase?

☐ A confusion

☐ B dedication

☐ C instruction

☐ D liberty

 Mark for Review

2. German-Dutch paleontologist Ralph von Koenigswald was the first to discover the fossilized remains of *Gigantopithecus blacki*, a gargantuan ape believed to have lived during the Pleistocene Epoch. Because the fossils were exclusively found in caves in southern China, many experts believe that the species was _____ that region—that is, anyone claiming to have found remains of *Gigantopithecus* elsewhere would be mistaken.

Which choice completes the text with the most logical and precise word or phrase?

☐ A restricted to

☐ B eliminated from

☐ C common in

☐ D unknown to

 Mark for Review

3. Computer scientist Ray Kurzweil _____ that although artificial intelligence will not displace human beings, it will undoubtedly become smarter than people within this generation. This possibility has been the domain of science fiction writers for decades, whose works explore the ramifications of just such a future.

Which choice completes the text with the most logical and precise word or phrase?

☐ A proves

☐ B requires

☐ C predicts

☐ D denies

 Mark for Review

4. In psychology, it's critical not to generalize from the results of studies in which the subjects are not representative of the larger population. The infamous Stanford Prison Experiment _____ this principle: the participants, whose behavior supposedly demonstrated the “human” tendency towards alarming aggression in authoritarian situations, were a handful of male college-age individuals from the same private university in California rather than a diverse sampling of subjects.

Which choice completes the text with the most logical and precise word or phrase?

☐ A illustrates

☐ B refutes

☐ C supersedes

☐ D critiques

 Mark for Review

5. Neurologists know that prosopagnosia—the _____ to recognize faces—involves a specific lesion in the brain and can be caused by disease or head injury. However, prominent author Dr. Oliver Sacks believes that this “face blindness” also has a definite genetic component.

Which choice completes the text with the most logical and precise word or phrase?

☐ A capability

☐ B incapacity

☐ C tendency

☐ D reluctance

 Mark for Review

6. The shark's competitive advantage in the oceanic ecosystem is principally due to electroreception, or the ability to detect electrical impulses. Marine biologists believe that this heightened _____ to electrical stimuli allows the shark to easily find its prey, for as fish swim through water, their movement produces minute electrical signals.

Which choice completes the text with the most logical and precise word or phrase?

☐ A allergy

☐ B sensitivity

☐ C indifference

☐ D aversion

 Mark for Review

7. The Voynich manuscript was written on vellum dating from the fifteenth century in a script that is not found in any other source. Since

cryptographers have yet to demonstrably decipher any portion of the text, the meaning and purpose of the Voynich manuscript remain _____ .

Which choice completes the text with the most logical and precise word or phrase?

☐ A enigmatic

☐ B venerable

☐ C multifarious

☐ D coherent



Mark for Review

8. It is commonly believed that, in the complex ecosystem of the Nile River in Africa, the crocodile and the Egyptian plover bird have formed an _____ relationship: the crocodile opens its mouth and keeps it open while the bird instinctively eats the food particles remaining in the crocodile's teeth, thus nourishing the bird while simultaneously promoting the crocodile's dental health.

Which choice completes the text with the most logical and precise word or phrase?

☐ A interdependent

☐ B inexplicable

☐ C enthralling

☐ D inarticulate

 Mark for Review

9. The following text is from Herman Melville's 1924 short novel *Billy Budd* and pertains to Edward Vere, the captain of the ship on which Billy is sailing.

Captain the Honorable Edward Fairfax Vere, to give his full title, was a bachelor of forty or thereabouts, a sailor of distinction even in a time prolific of renowned seamen. Though allied to the higher nobility, his advancement had not been altogether owing to influences connected with that circumstance. He had seen much service, been in various engagements, always acquitting himself as an officer mindful of the welfare of his men, but never tolerating an infraction of discipline; thoroughly versed in the science of his profession, and intrepid to the verge of temerity, though never injudiciously so.

According to the text, what is true of Captain Vere?

☐ A He dislikes many of the men who serve under him.

☐ B He is proud of his aristocratic background.

☐ C He is a capable and evenhanded naval officer.

☐ D He prefers navy life to life outside the navy.

 Mark for Review

10. "I Remember, I Remember" is an 1844 poem by Thomas Hood. The poem conveys the speaker's sadness that his life as an adult does not compare favorably to his childhood: _____.

Which quotation from the poem most effectively illustrates the claim?

A “The lilacs where the robin built, / And where my brother
set / The laburnum on his birthday,— / The tree is living
yet!”

B “I remember, I remember, / The house where I was born, /
The little window where the sun / Came peeping in at
morn.”

C “I remember, I remember, / The roses, red and white, / The
vi’lets, and the lily-cups, / Those flowers made of light!”

D “It was a childish ignorance, / But now ‘tis little joy / To
know I’m farther off from heav’n / Than when I was a
boy.”

 Mark for Review

11. *Dracula* is an 1897 novel by Bram Stoker. In the story, English lawyer Jonathan Harker has traveled to Transylvania to conduct business with Count Dracula at his castle. In his journal, Harker conveys his belief that he has become Dracula’s prisoner: _____.

Which quotation from Jonathan Harker’s journal most effectively illustrates the claim?

A “What manner of man is this, or what manner of creature,
is it in the semblance of man? I feel the dread of this
horrible place overpowering me.”

B “My lamp seemed to be of little effect in the brilliant
moonlight, but I was glad to have it with me, for there was
a dread loneliness in the place which chilled my heart and
made my nerves tremble.”

☐ C “I start at my own shadow, and am full of all sorts of horrible imaginings. God knows that there is ground for my terrible fear in this accursed place!”

☐ D “I rushed up and down the stairs, trying every door and peering out of every window I could find, but after a little the conviction of my helplessness overpowered all other feelings.”

 Mark for Review

12. “In Flanders Fields” is a 1915 poem written by Lieutenant-Colonel John McCrae, a Canadian military officer who died three years later in World War I. The poem is meant to be a plea towards others to join the war effort, as is evident by the following lines: _____.

Which quotation from “In Flanders Fields” most effectively illustrates the claim?

☐ A “Loved and were loved and now we lie / In Flanders fields”

☐ B “In Flanders fields the poppies blow / Between the crosses row on row”

☐ C “To you from failing hands we throw / The torch; be yours to hold it high”

☐ D “We are the dead. Short days ago / We lived, felt dawn, saw sunset glow”

 Mark for Review

13. The curator of a museum claims that a dress in his possession was worn by the wife of one of Lincoln's generals at the presidential inauguration in 1865. Radiocarbon dating, which dates organic material with an error range of about thirty years in either direction, was performed on the sleeves of the dress, revealing that they date back to the 1975–2005 period. If both the curator's claim and the radiocarbon dating analysis are correct, that would suggest that _____.

Which choice most logically completes the text?

☐ A the dress was made sometime between 1835 and 1895 and then damaged sometime after 1975.

☐ B vintage dresses are more commonly recovered from the late twentieth and early twenty-first centuries than from the mid-nineteenth century.

☐ C over one hundred years after the dress was made, its sleeves were replaced.

☐ D the dress was made from material different from that used for most dresses in the nineteenth century.

 Mark for Review

14. In the early 1900s, paleontologists largely believed that there were no undocumented prehistoric aquatic species that had survived to the present day because it would be impossible for such a species to have enough animals to sustain a breeding population while escaping detection in the modern era. However, a coelacanth, a large lobe-finned fish universally believed by scientists to have gone extinct sixty-six million years ago, was found off the coast of South Africa as recently as 1938. This event may suggest that _____.

Which choice most logically completes the text?

☐ A fewer coelacanths are required to sustain a breeding population than was previously thought.

☐ B it is possible for a prehistoric species to go undiscovered for longer than expected.

☐ C the scientists who determined that the coelacanth was extinct ignored critical evidence.

☐ D the same environmental conditions that eliminated the dinosaurs nearly killed off the coelacanths.

 Mark for Review

15. The *door-in-the-face* technique involves initially making an outrageous or unappealing request or offer, which the other person is highly likely to refuse, then following up with a more reasonable one. The subject is more likely to look favorably upon this second request or offer because it seems acceptable compared to the initial proposition. So, if an employee wants the best raise in annual salary from her boss that she can get, she might succeed by asking for a _____.

Which choice most logically completes the text?

☐ A 50% raise, then asking for a 5% raise.

☐ B 3% raise, then asking for a 2% raise.

☐ C 10% raise, then asking for a 50% raise.

☐ D 3% raise, then asking for a 3% raise again.

 Mark for Review

16. The North American Free Trade Agreement (NAFTA) was an agreement among the United States, Canada, and Mexico that was in effect between 1994 and 2020. During this time, the number of manufacturing jobs in the United States and Canada declined, but the total number of manufacturing jobs in the countries covered by NAFTA increased. This suggests that, between 1994 and 2020, _____.

Which choice most logically completes the text?

☐ A the number of manufacturing jobs in Mexico increased by a greater amount than the combined decreases in the United States and Canada.

☐ B NAFTA made it more difficult for manufacturers to establish factories in the United States and Canada.

☐ C the cost of manufacturing goods in the area covered by NAFTA decreased.

☐ D complex goods, such as automobiles and electronics, were increasingly manufactured in the United States, Canada, and Mexico.

 Mark for Review

17. American chef Alice Waters is well-known for opening the restaurant Chez Panisse, which _____ the farm-to-table movement by serving local and seasonal food.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A originating

☐ B to originate

☐ C having originated

☐ D originated

 Mark for Review

18. American activists Dolores Huerta and Cesar Chavez founded the National Farm Workers Association in 1962 to defend the rights of farm workers through nonviolent organizing tactics, such as marches and boycotts. _____ organization merged with the Agricultural Workers Organizing Committee, led by Larry Itliong, to form United Farm Workers, a labor union that advocates on behalf of farm workers across the U.S.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A Its

☐ B Their

☐ C It's

☐ D They're

 Mark for Review

19. Researchers at the University of York found that people who are highly individualistic feel less connected to the natural world and engage in fewer activities to improve the environment; however,

engaging with nature through activities such as walking and bird-watching can reconnect _____ to the natural world and encourage environmentally-friendly behaviors.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A it

☐ B you

☐ C one

☐ D them



Mark for Review

20. National flags are designed to best represent and symbolize the individual _____ when countries share a history or culture, their flags are designed to look similar, thus creating a flag family that shares colors, shapes, or other elements.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A country but

☐ B country,

☐ C country

☐ D country, but



Mark for Review

21. Scientists at the University of Illinois and the University of Lancaster observed that plants under very bright sunlight enter a protective mode for several minutes, during which they stop photosynthesizing and growing. If the crops were genetically modified to have a shorter time in protective mode, _____ The scientists resolved to find out.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A could the crop yield increase?

☐ B the crop yield could increase?

☐ C the crop yield could increase.

☐ D could the crop yield increase.

 Mark for Review

22. Take Our Daughters and Sons to Work Day originally started as a day focused on engaging girls with the workforce, Take Our Daughters to Work Day. On the national day, the fourth Tuesday in April, parents and caregivers go to work with their children; shadowing their parents or caregivers _____ children real-world experience and ideas for potential future careers.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ A offer

☐ B have offered

☐ C are offering

☐ offers

 Mark for Review

23. In order to allow olive ridley turtles to lay eggs on Versova Beach in Mumbai, community activist Afroz Shah organized a large group of volunteers to remove over 11 million pounds of trash. The beach now allows community members to connect with the natural world and _____ a healthy habitat for olive ridley turtles to use after a twenty-year absence.

Which choice completes the text so that it conforms to the conventions of Standard English?

☐ provided

☐ providing

☐ provides

☐ provide

 Mark for Review

24. Yoga is an ancient discipline from India that aims to combine physical fitness with mental and spiritual control and calm and has expanded to become popular with many different cultures. _____ yoga is shifting into different forms to allow a wider range of people to participate. For example, accessible yoga provides opportunities for those with physical disabilities to access the health and mental benefits of the practice.

Which choice completes the text with the most logical transition?

☐ A Nevertheless,

☐ B Similarly,

☐ C Thus,

☐ D Currently,

 Mark for Review

25. Scientists often disagree about what traits to use to place newly discovered species in the tree of life and debate different ways to organize evolutionary relationships. *Chimerarachne yingi*, _____ is an extinct arachnid species that is sometimes placed near modern spiders based on its acquisition of silk-spinning organs or near other arachnids based on its loss of a tail.

Which choice completes the text with the most logical transition?

☐ A as a result,

☐ B in comparison,

☐ C for example,

☐ D still,

 Mark for Review

26. In 2011, a seismometer detected seismic activity from a magnitude 8.9 earthquake and automatically cut the power to all 30 bullet trains in Japan, potentially avoiding mass architectural damage to the tracks. _____ the cut to the power prevented citizens from being caught

in a dangerous location during the earthquake and allowed riders to seek shelter.

Which choice completes the text with the most logical transition?

☐ A In addition,

☐ B In comparison,

☐ C For example,

☐ D Specifically,

 Mark for Review

27. While researching a topic, a student has taken the following notes:

- The Endangered Species Act (ESA) was enacted in 1973 to recover species and prevent extinction.
- A species is listed under the ESA when it's determined that the species needs protection and delisted when the population has recovered.
- Only 54 of the over 1,000 listed species have been delisted from the ESA, raising concerns about the effectiveness of the ESA.
- Erich Eberhard, David Wilcove, and Andrew Dobson conducted an analysis of population trends of species listed under the ESA.
- They found that most species had to wait multiple years before being listed and by then their populations were already so low that recovery was much more difficult.

The student wants to make a generalization about the kind of study conducted by Eberhard, Wilcove, and Dobson. Which choice most

effectively uses relevant information from the notes to accomplish this goal?

- ☐ A Scientists have analyzed population trends to find out the impact of legal protections in the realm of conservation.
- ☐ B Species listed under the ESA have low population levels when they are listed.
- ☐ C Only 54 once-listed species have been delisted; many more species have not recovered and are still listed.
- ☐ D Based on an analysis of population trends, Eberhard, Wilcove, and Dobson found that species listed under the ESA have very small populations when listed.

S T O P

**If you finish before time is called, you may check your work on this module only.
Do not turn to any other module in the test.**

Section 1, Module 2—Harder: Reading and Writing

SAT Prep Test 2—Reading and Writing

Module 2—Harder

Turn to Section 1 of your answer sheet ([this page](#)) to answer the questions in this section.

DIRECTIONS

The questions in this section address a number of important reading and writing skills. Each question includes one or more passages, which may include a table or graph. Read each passage and question carefully, and then choose the best answer to the question based on the passage(s).

All questions in the section are multiple-choice with four answer choices. Each question has a single best answer.

 Mark for Review

1. Dutch philosopher Baruch Spinoza argued as part of his rejection of dualism that all things, living or not, have the inclination to continue to exist and enhance themselves, a property he named “conatus.” All things, he believed, had the tendency to _____ and would only cease to be if acted upon by outside forces.

Which choice completes the text with the most logical and precise word or phrase?

☐ A deteriorate

☐ B perish

☐ C persevere

☐ D disappear